NOBEL PRIZE IN PHYSICS 2016

the role of topology in condensed matter

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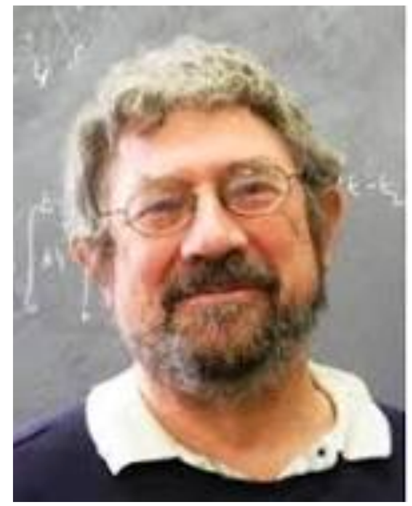
Santiago de Compostela, 1/02/2017

"for theoretical discoveries of topological phase transitions and topological phases of matter"

Thouless



Kosterlitz

Haldane

$\frac{1}{4}$ 

KT transition

$\frac{1}{4}$ 

KT transition

$\frac{1}{4}$ 

gapped
spin chains

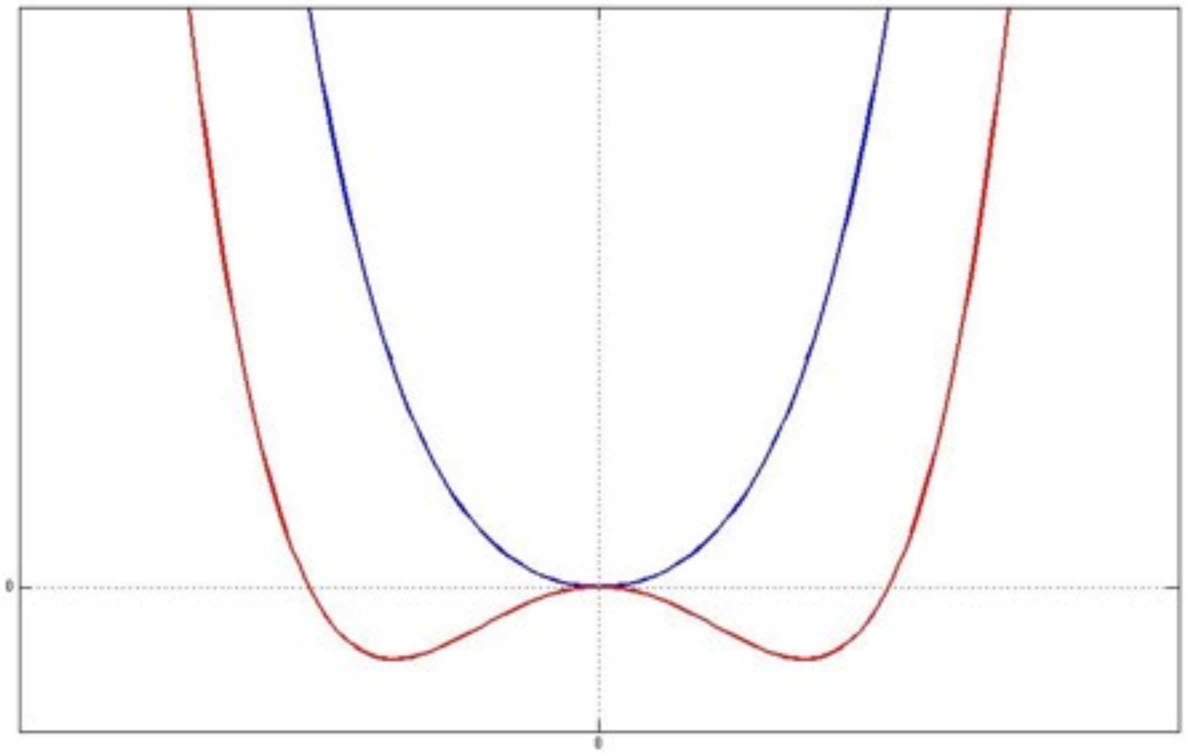
$\frac{1}{4}$ 

TKNN invariant
Hall conductivity

Thors H. Hansson happy explaining
topology with the committee’s breakfast

Why he can use pastries with holes to explain topology?

KT transition



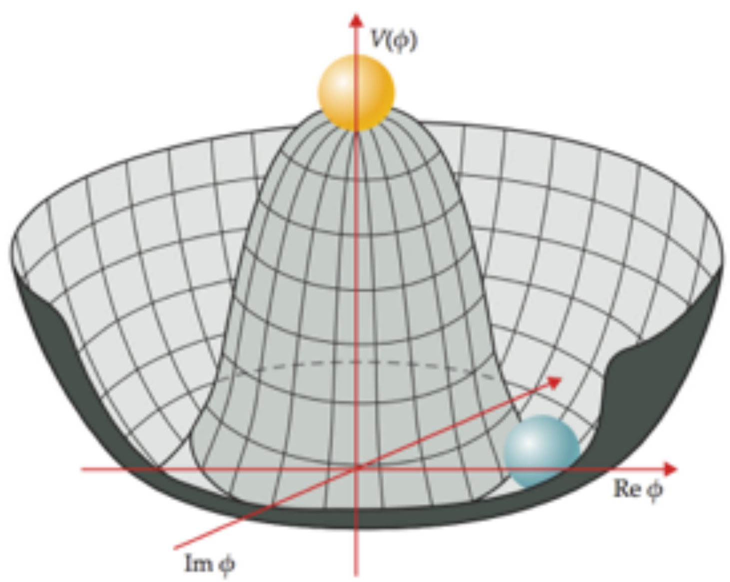
ρ

$a < 0$ $\qquad \rho = 0$

$a > 0$ $\qquad \rho > 0$

$$\mathcal{V}[\phi] = a\phi^2 + b\phi^4$$

KT transition

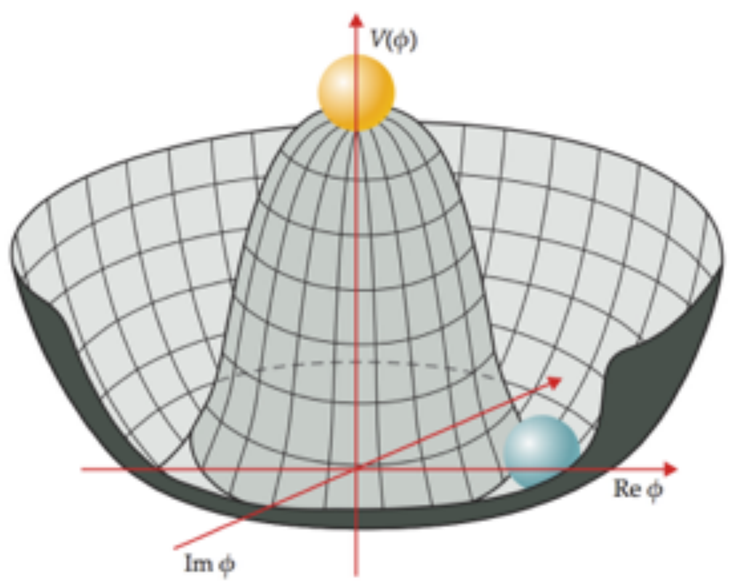


$a < 0$ $\rho = 0$

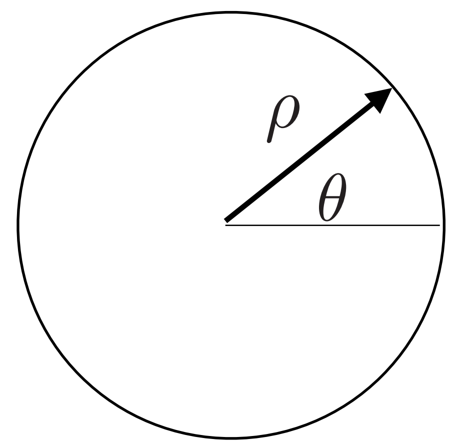
$a > 0$ $\rho > 0$

$$\mathcal{V}[\boldsymbol{\phi}] = a\boldsymbol{\phi}^* \cdot \boldsymbol{\phi} + b(\boldsymbol{\phi}^* \cdot \boldsymbol{\phi})^2$$

KT transition



$$U \sim \int d^D \boldsymbol{r} \boldsymbol{\phi}^2$$



$$\phi_1(\boldsymbol{r}) + i\phi_2(\boldsymbol{r}) = \rho(\boldsymbol{r}) e^{i\theta(\boldsymbol{r})}$$

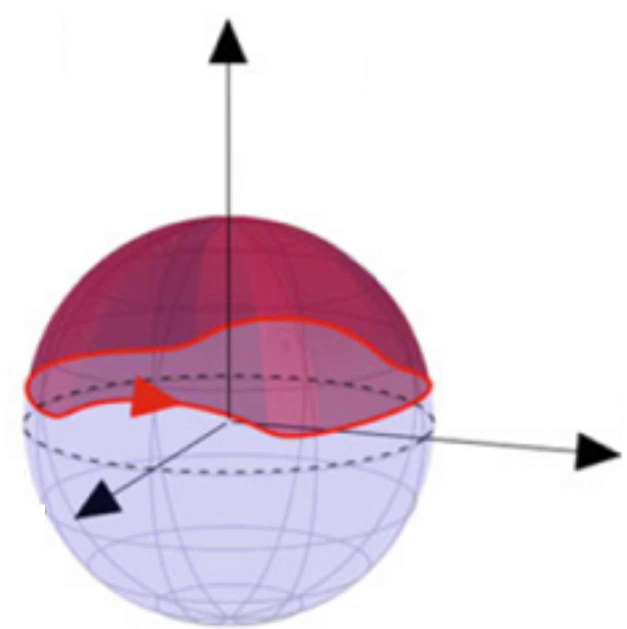
$a < 0 \qquad \rho = 0$

$a > 0 \qquad \rho > 0$

$$\mathcal{V}[\boldsymbol{\phi}] = a\boldsymbol{\phi}^* \cdot \boldsymbol{\phi} + b(\boldsymbol{\phi}^* \cdot \boldsymbol{\phi})^2$$

$$\boldsymbol{S} = \langle S \rangle (\boldsymbol{r}) \cdot \boldsymbol{n}(\boldsymbol{r})$$

$$\boldsymbol{n}^2 = 1$$

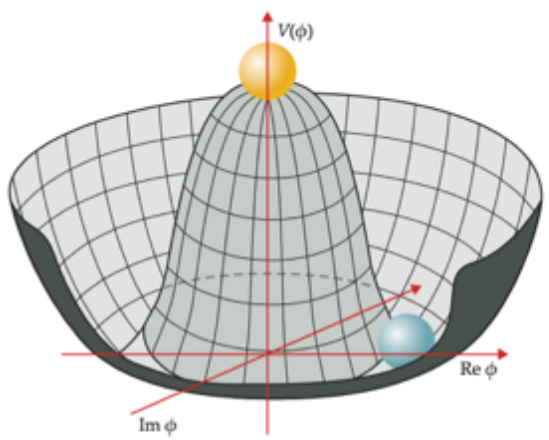


KT transition

$$\phi_1(\boldsymbol{r}) + i\phi_2(\boldsymbol{r}) = \rho(\boldsymbol{r})e^{i\theta(\boldsymbol{r})}$$

$$U \sim c \int d^D\boldsymbol{r}(\boldsymbol{\nabla}\theta)^2$$ Goldstone boson

$$U \sim \int d^D\boldsymbol{r}(\boldsymbol{\nabla}\delta\rho)^2 + m^2\delta\rho^2$$ massive amplitude mode



$$\langle\boldsymbol{\phi}\rangle \simeq \rho - T\frac{\rho}{2}\langle\theta(\boldsymbol{x})\theta(\boldsymbol{x})\rangle$$

$$\langle\theta(\boldsymbol{x})\theta(\boldsymbol{x})\rangle \sim \int d^D\boldsymbol{k}\frac{1}{\boldsymbol{k}^2}$$

in 2 dimensions, the integral diverges in the thermodynamic limit

KT transition

$$\phi_1(\boldsymbol{r}) + i\phi_2(\boldsymbol{r}) = \rho(\boldsymbol{r})e^{i\theta(\boldsymbol{r})}$$

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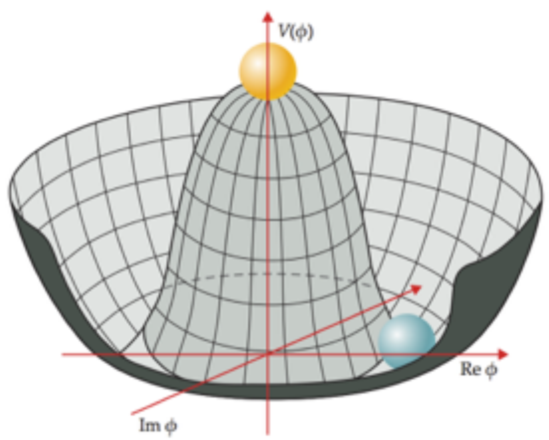


$$\langle \boldsymbol{\phi} \rangle \simeq \rho - T\frac{\rho}{2} \langle \theta(\boldsymbol{x})\theta(\boldsymbol{x}) \rangle$$

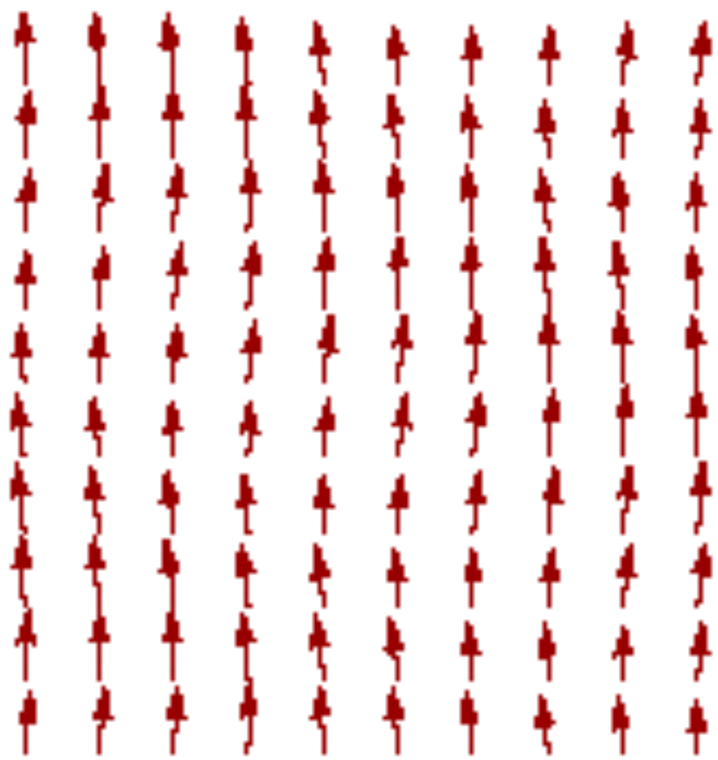
$$\langle \theta(\boldsymbol{x})\theta(\boldsymbol{x}) \rangle \sim \int d^D \boldsymbol{k} \frac{1}{\boldsymbol{k}^2}$$

in 2 dimensions, the integral diverges in the thermodynamic limit

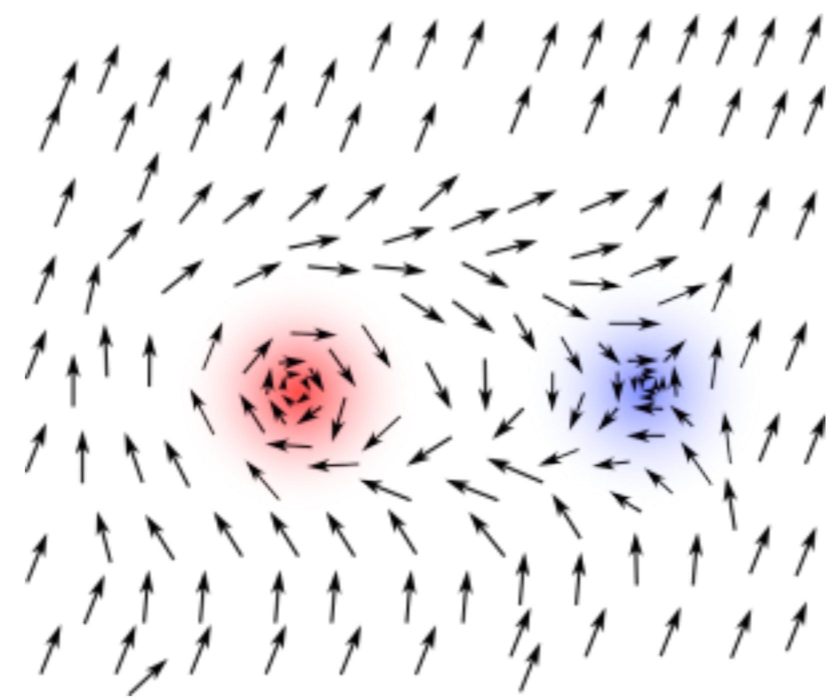
Mermin-Wagner Theorem

KT transition

where appears that the phase is a phase?



vs.



$$\vec{\phi} = \vec{\nabla}\theta$$

$$\vec{\nabla} \times \vec{\phi} = \vec{\nabla} \times \vec{\nabla}\theta = 0$$

winding number!

$$\oint d\vec{l} \cdot \vec{\phi} = \int d^2 r \vec{e}_3 \cdot \vec{\nabla} \times \vec{\phi} = 2\pi \sum_j n_j$$

$$\vec{\phi} = \vec{\nabla}\theta + \vec{\nabla} \times (\vec{e}_3 \psi)$$

$$\vec{\nabla} \times \vec{\phi} = \vec{e}_3 \nabla^2 \psi = 2\pi \sum_j n_j \delta(\boldsymbol{r} - \boldsymbol{R}_j)$$

KT transition

where appears that the phase is a phase?

$$\vec{\phi} = \vec{\nabla}\theta + \vec{\nabla} \times (\vec{e}_3\psi)$$

$$\vec{e}_3 \cdot \vec{\nabla} \times \vec{\phi} = \nabla^2\psi = 2\pi \sum_j n_j \delta(\boldsymbol{r} - \boldsymbol{R}_j)$$

$$U \sim \int d^2\boldsymbol{r}\boldsymbol{\phi}^2 \quad \Longrightarrow \quad U \sim \int d^2\boldsymbol{r}(\vec{\nabla}\theta)^2 - \sum_{i,j} n_i n_j C(\vec{R}_i - \vec{R}_j)$$

KT transition

where appears that the phase is a phase?

$$\vec{\phi} = \vec{\nabla}\theta + \vec{\nabla} \times (\vec{e}_3 \psi)$$

$$\vec{e}_3 \cdot \vec{\nabla} \times \vec{\phi} = \nabla^2 \psi = 2\pi \sum_j n_j \delta(\boldsymbol{r} - \boldsymbol{R}_j)$$

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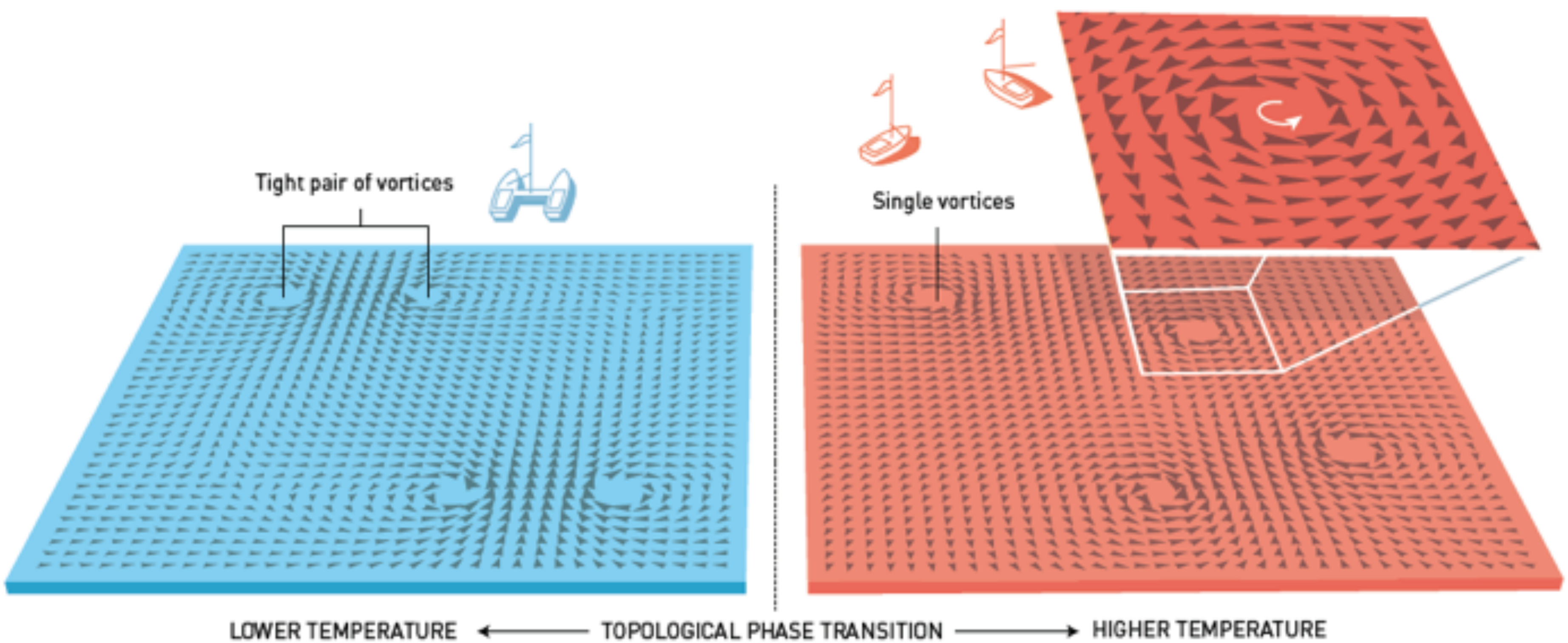
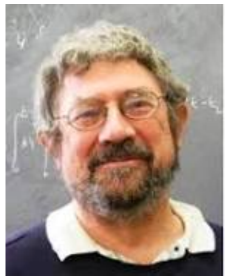
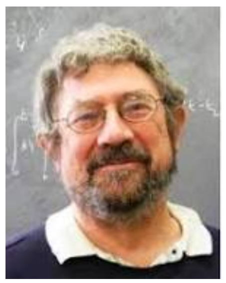




KT transition

where appears that the phase is a phase?

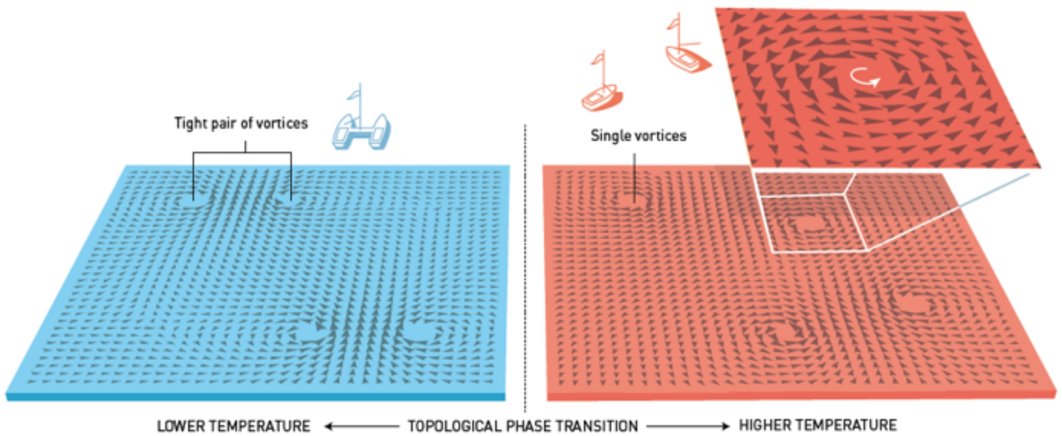


$$U \sim \int d^2\boldsymbol{r}(\vec{\nabla}\theta)^2 - \sum_{i,j} n_i n_j C(\vec{R}_i - \vec{R}_j)$$

$$\mathcal{Z}_V = \sum_N \sum_{\boldsymbol{R}_j} \frac{1}{(N!)^2} \int \Pi_{j=1}^{2N} d^2\boldsymbol{R}_j e^{-\beta U_V}$$

$$\mathcal{Z}_{sw} = \sum_n e^{-\beta U_{sw}}$$

$$\mathcal{Z} \sim \mathcal{Z}_{sw}\mathcal{Z}_V$$



$$C(\vec{R}) \sim \log(|\vec{R}|)$$

$$N_{sites} \sim (R/a)^2$$

$$U \simeq \kappa \log(R/a)$$

$$S \simeq 2k_B \log(R/a)$$

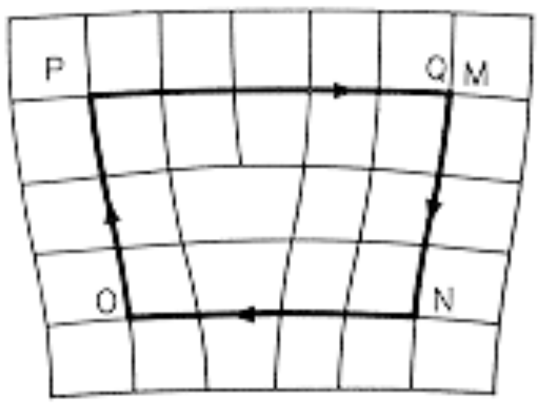
$$F = U - TS$$

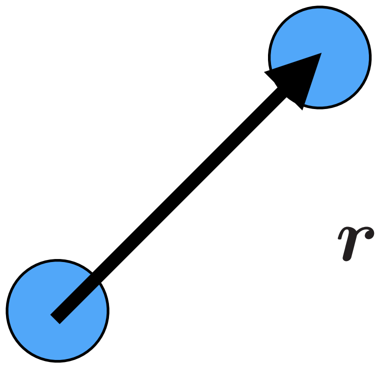
$$T_c = E/S = \kappa/2k_B$$

M Kosterlitz, DJ Thouless. J. Phys. C: Solid State Phys. 6 1181 (973)

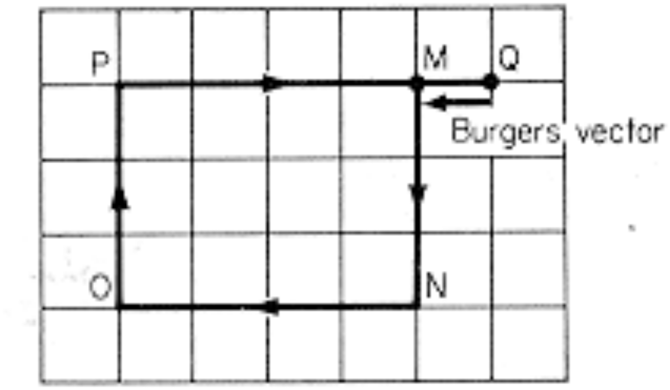
KT transition

two dimensional lattice melting





$$\boldsymbol{r} \to \boldsymbol{r} + \boldsymbol{u}(\boldsymbol{r})$$



$$\boldsymbol{u}(\boldsymbol{r}) = \bar{\boldsymbol{u}} + \boldsymbol{v}$$

$$\oint \boldsymbol{v} \cdot d\boldsymbol{r} = \boldsymbol{b}$$

$$\oint \bar{\boldsymbol{u}} \cdot d\boldsymbol{r} = 0$$

$$\Delta U \sim \int d^2\boldsymbol{x} d^2\boldsymbol{y} \rho(\boldsymbol{x}) C(\boldsymbol{x} - \boldsymbol{y}) \rho(\boldsymbol{y})$$

KT transition

2D superconductor

$$U \sim \int d^2\boldsymbol{r} \frac{1}{2m} |\partial\psi|^2 + a|\psi|^2 + b|\psi|^4$$

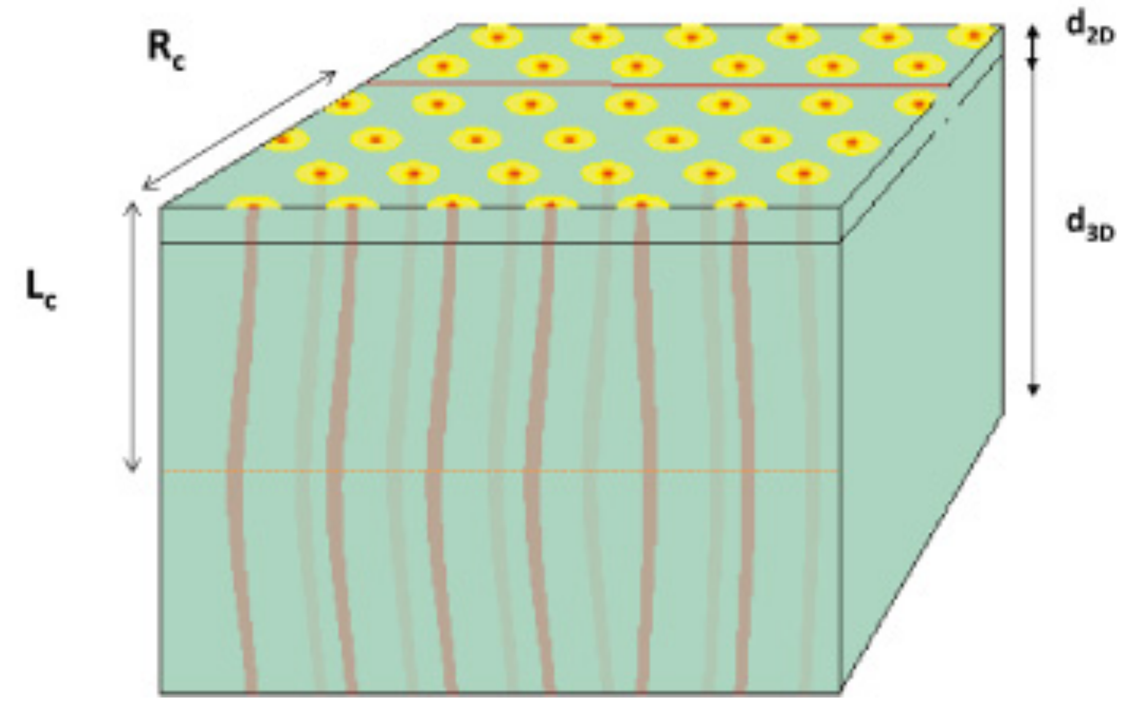
$$\psi = \sqrt{\rho} e^{i\theta(\boldsymbol{r})}$$

$$U \sim \int d^2\boldsymbol{r} \partial^a \theta(\boldsymbol{r}) \partial_a \theta(\boldsymbol{r})$$

3D lattice $d_{3D} >> L_c$

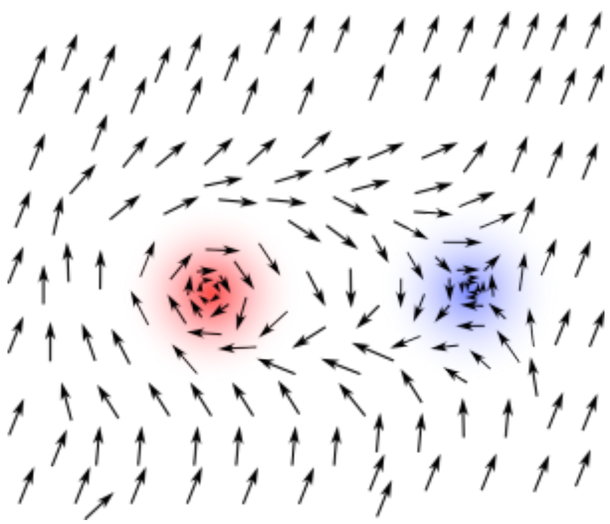
2D lattice $d_{2D} << L_c$

$$U \sim \int d^2\boldsymbol{r} (\vec{\nabla}\theta)^2 - \sum_{i,j} n_i n_j C(\vec{R}_i - \vec{R}_j)$$



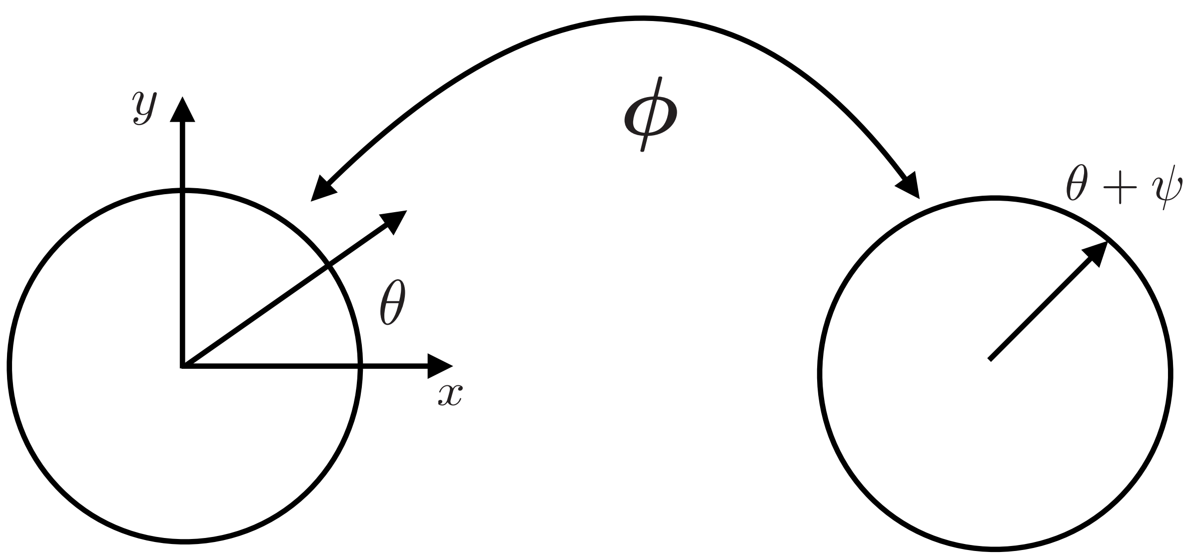
KT transition

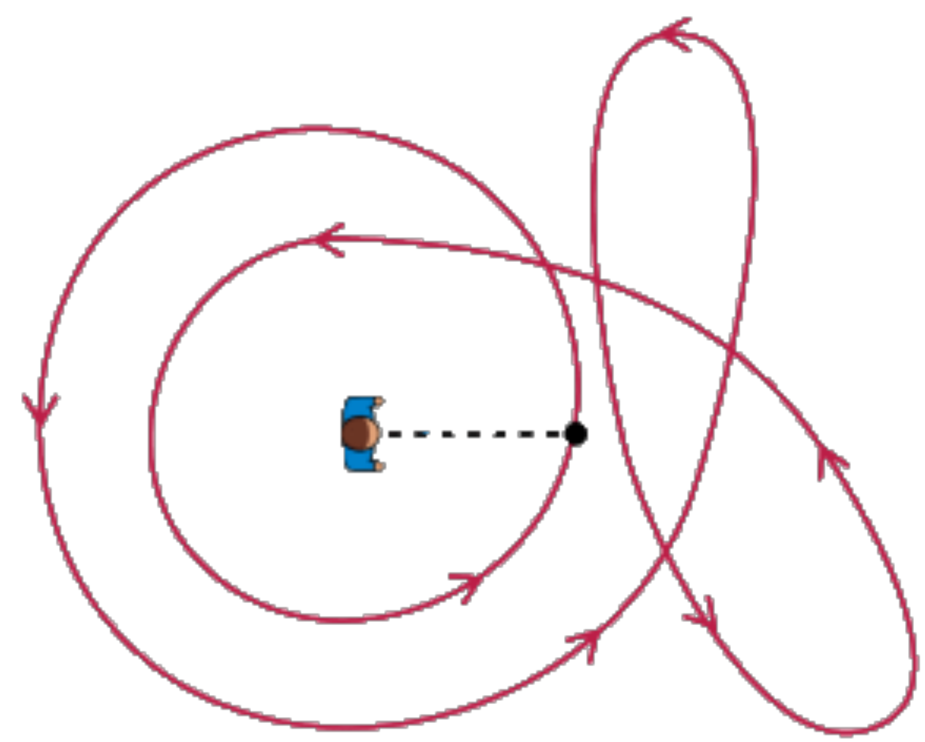
where appears that the phase is a phase?



$$\oint d\vec{l} \cdot \vec{\phi} = \int d^2 r \vec{e}_3 \cdot \vec{\nabla} \times \vec{\phi} = 2\pi \sum_j n_j$$

winding number



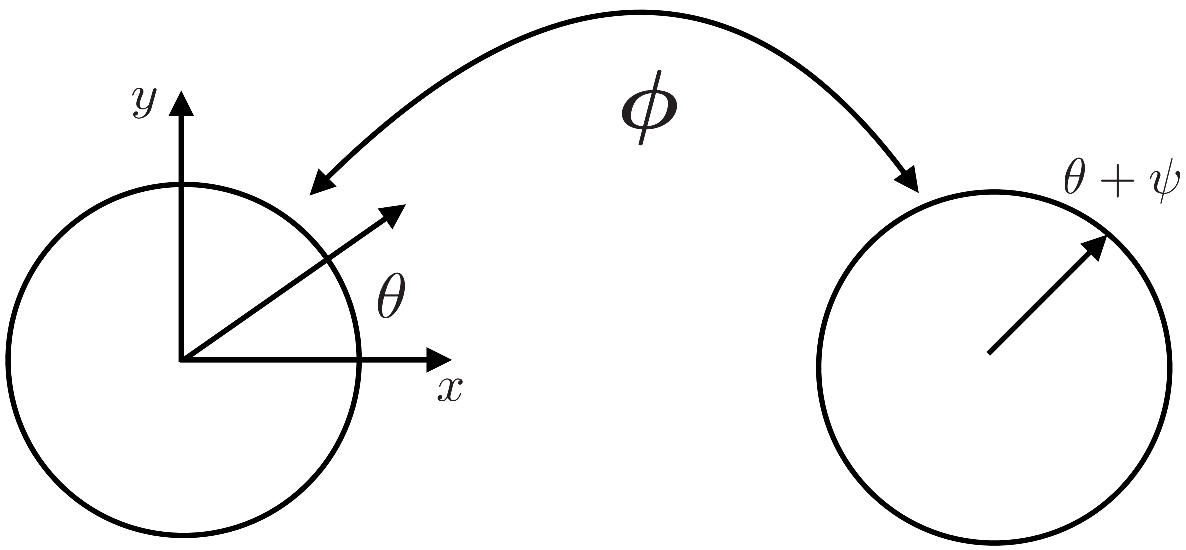


$$\vec{\phi} = \vec{\nabla}\theta + \vec{\nabla} \times (\vec{e}_3 \psi)$$

winding

“bouncing”

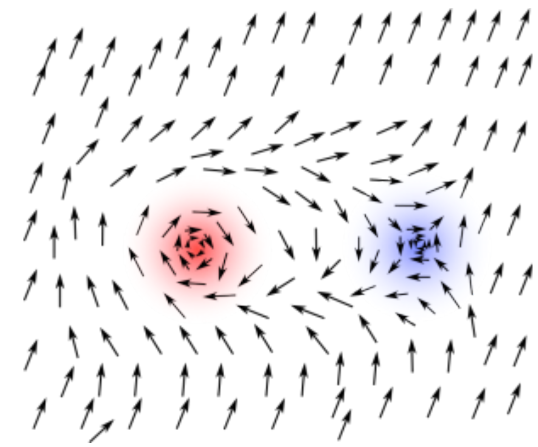
Topology

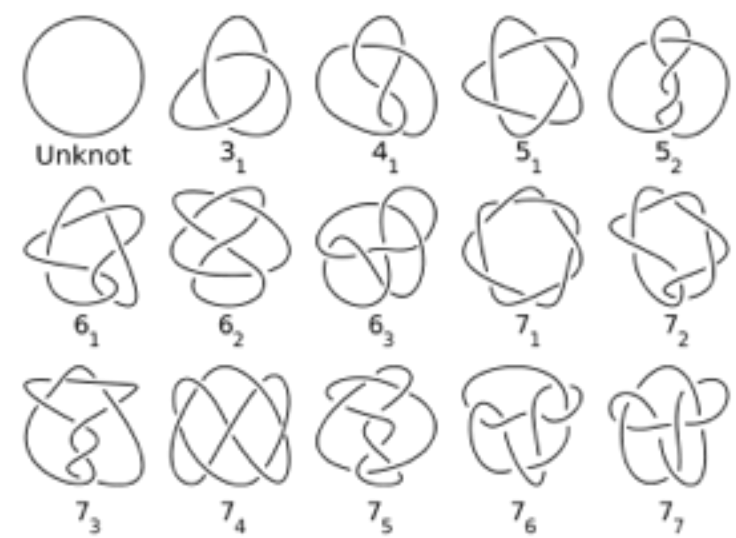


$$\phi : S^1 \to S^1$$

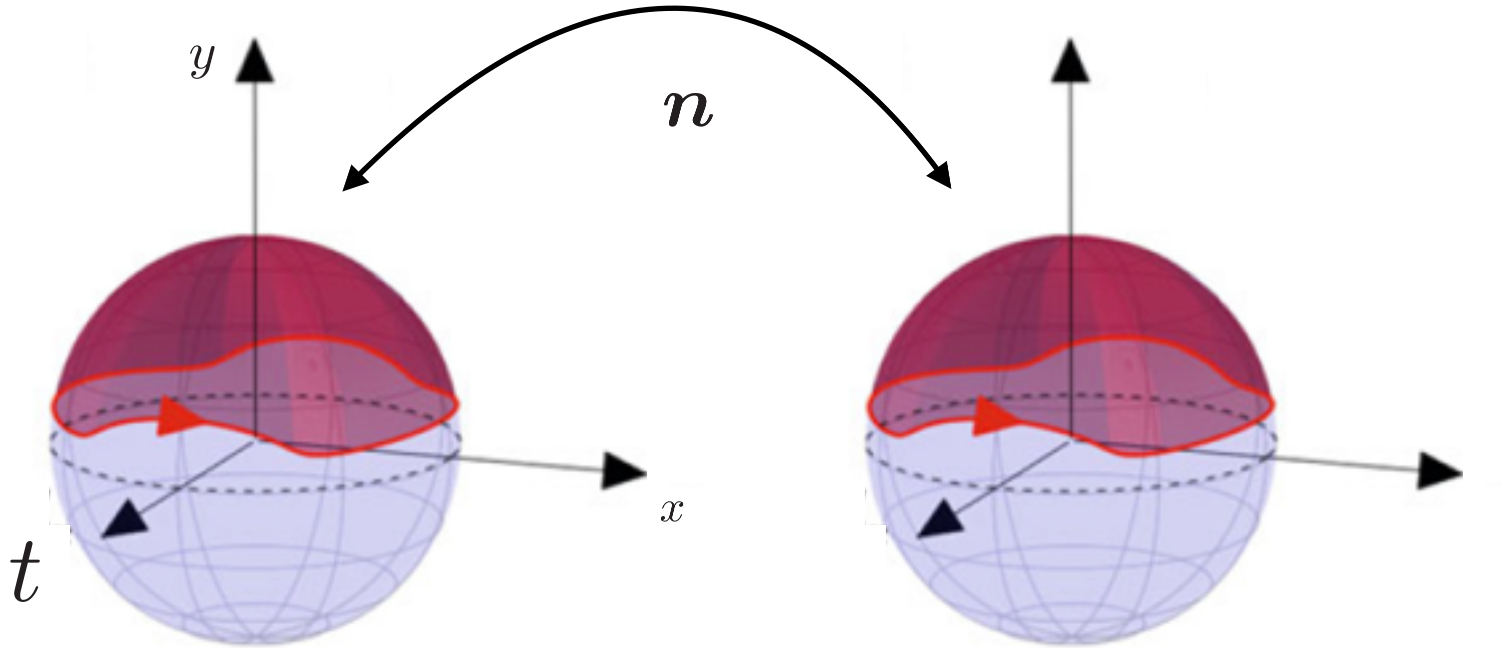
$$\pi_1(S^1) = \mathbb{Z}$$

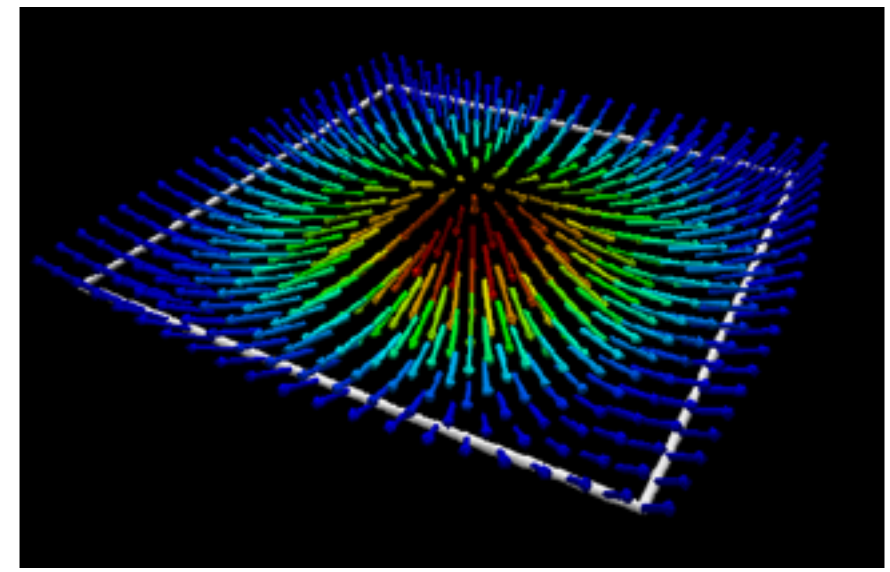
all types of “mappings” are classified according to its winding number





Topology





$$H = J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\boldsymbol{n} : S^d \to S^2$$

$$\pi_1(S^2) = 0$$

$$\pi_2(S^2) = \mathbb{Z}$$

$$\pi_3(S^2) = \mathbb{Z}$$

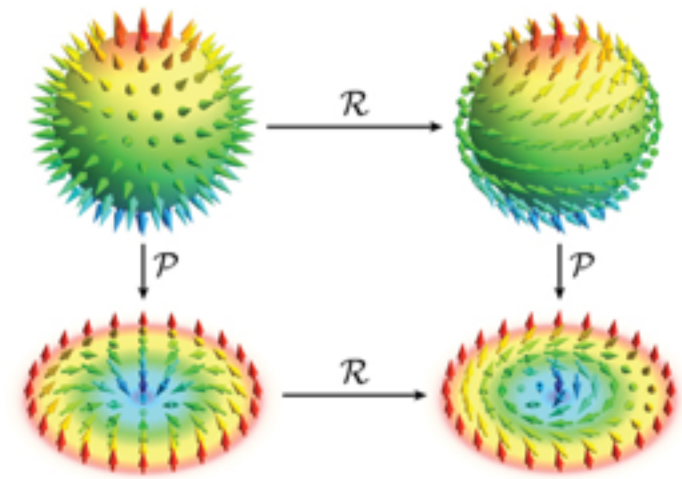
$$\mathcal{Q} = \int d^2\boldsymbol{r}(\partial_1 \boldsymbol{n} \times \partial_2 \boldsymbol{n}) \cdot \boldsymbol{n}$$

skyrmion number

Topology

A non-linear field theory

By T. H. R. Skyrme
Atomic Energy Research Establishment, Harwell



Metastable states of two-dimensional isotropic ferromagnets

A. A. Belavin and A. M. Polyakov

$$F_{\mu\nu} = -\frac{1}{e|\boldsymbol{r}|^3}\epsilon_{\mu\nu a} r_a$$

$$A_\mu = g^{-1}\partial_\mu g$$

MAGNETIC MONOPOLES IN UNIFIED GAUGE THEORIES

G. 't HOOFT
CERN, Geneva

$$g = e^{i\boldsymbol{\sigma}\cdot\boldsymbol{n}(x)}$$

Received 31 May 1974

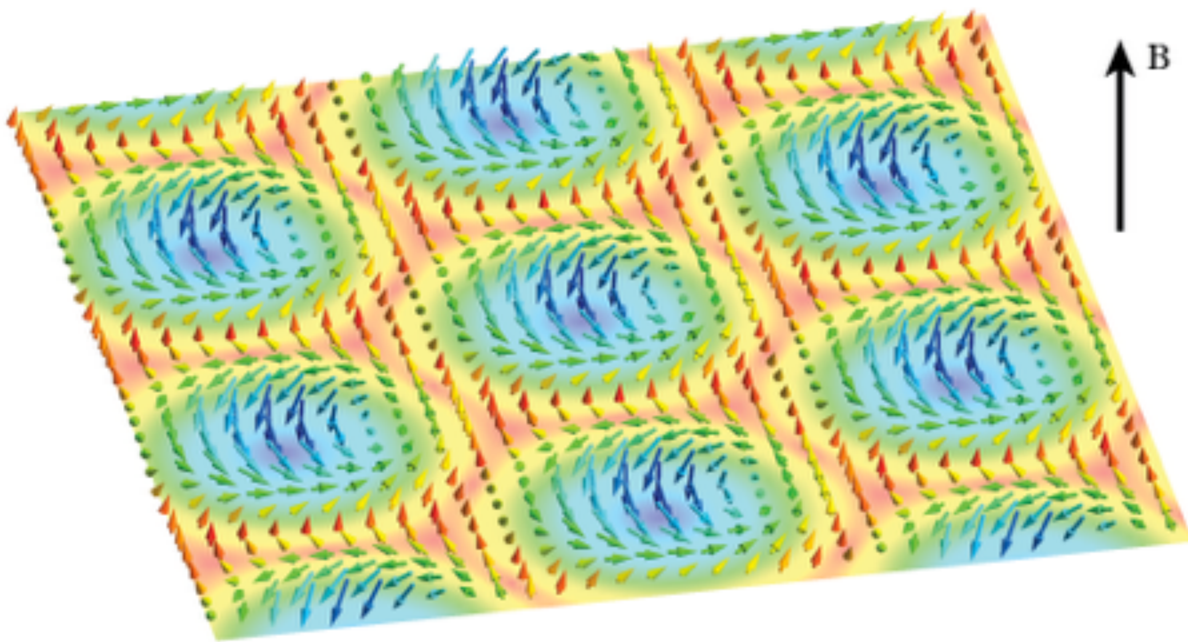
VOLUME 50, NUMBER 15 PHYSICAL REVIEW LETTERS 11 APRIL 1983

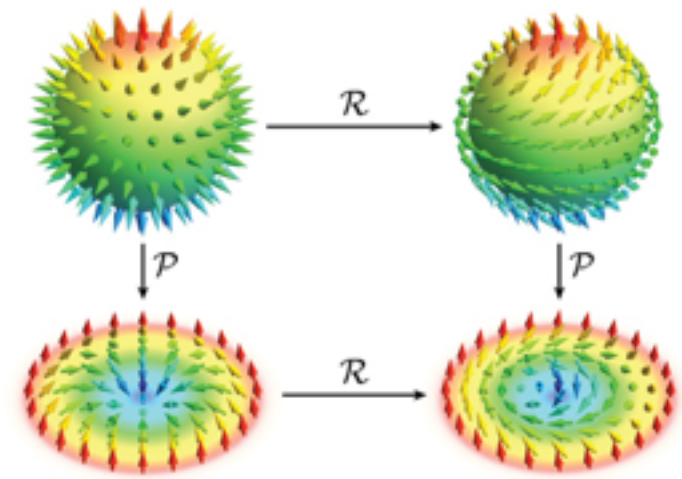
Nonlinear Field Theory of Large-Spin Heisenberg Antiferromagnets: Semiclassically Quantized Solitons of the One-Dimensional Easy-Axis Néel State

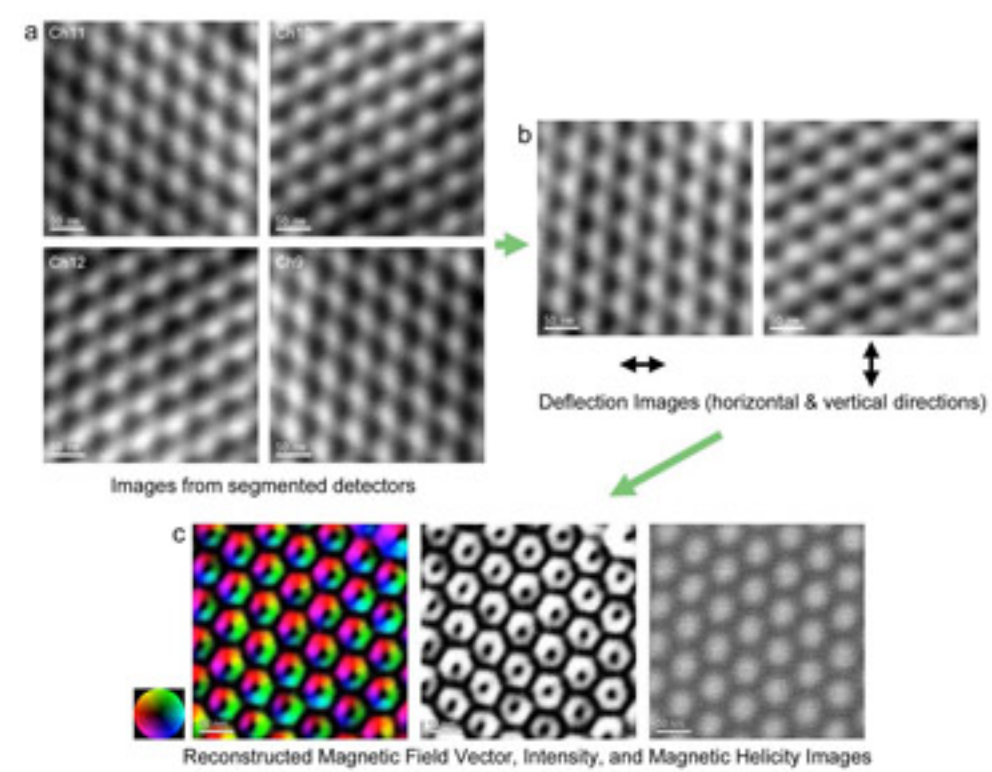
F. D. M. Haldane
Department of Physics, University of Southern California, Los Angeles, California 90089

(Received 31 January 1983)

Topology



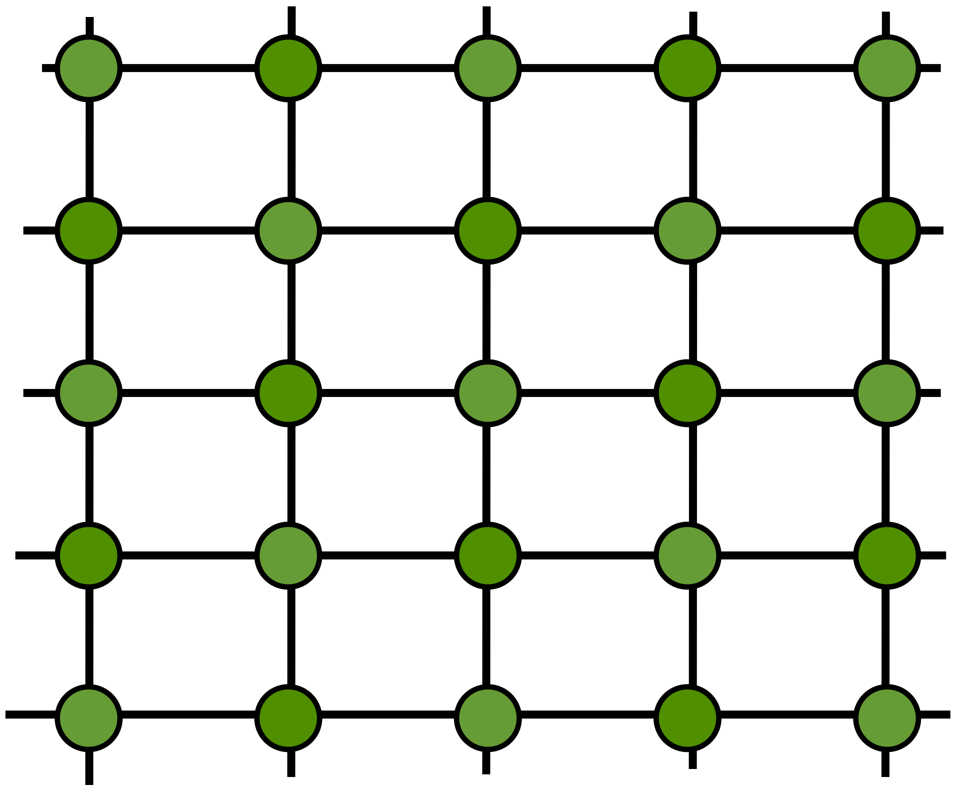




skyrmion lattices in chiral magnets

Topology in electronic systems

Bloch theorem



$$H = H_0 + V(\mathbf{r})$$

$$V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a})$$

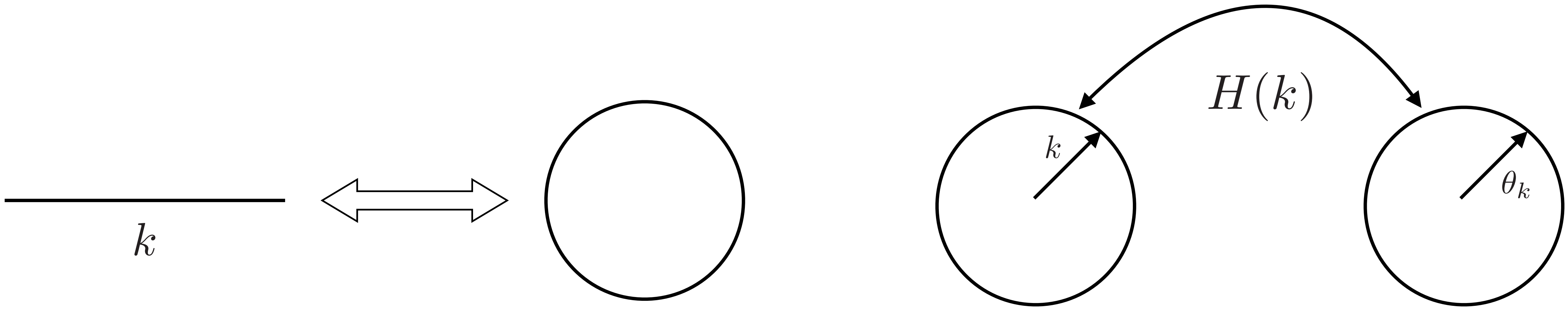
$$\boxed{|\psi_n(\mathbf{r})\rangle = e^{i\mathbf{k}\cdot\mathbf{r}}|u_{n,\mathbf{k}}\rangle}$$

each quantum state can be labeled by a vector $\mathbf{k}$

the energies form bands and they are periodic functions of $\mathbf{k}$

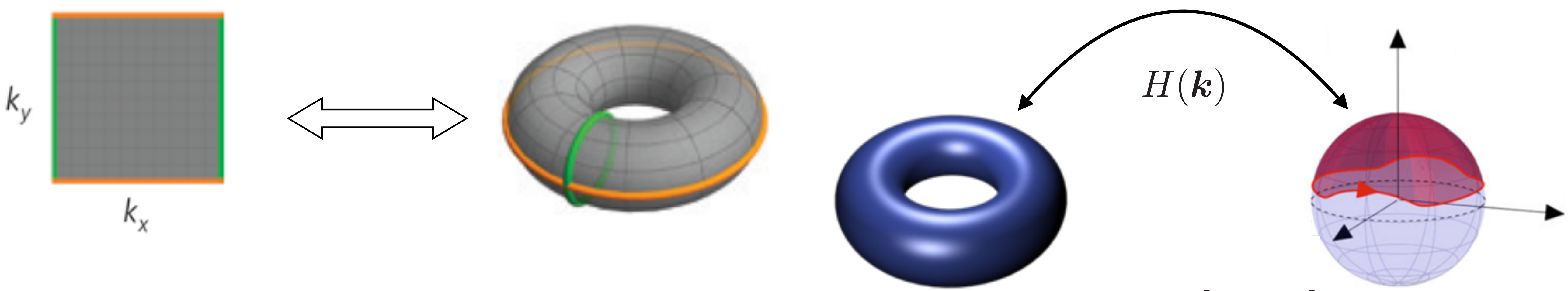
$$E_n(\mathbf{k}) = E_n(\mathbf{k} + \mathbf{G})$$

Topology in electronic systems



$$H(k) : S^1 \to S^1$$

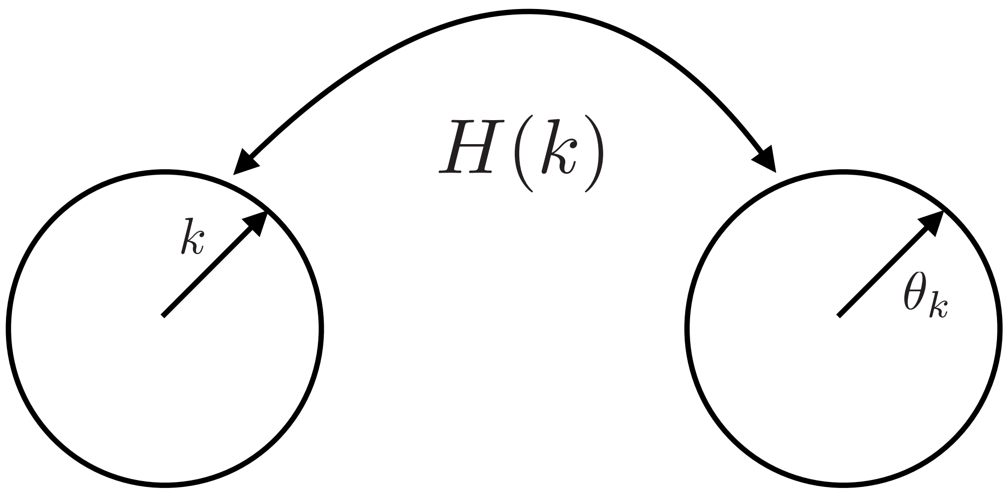
$$\pi_1(S^1) = \mathbb{Z}$$



$$H(\boldsymbol{k}) : T^2 \to S^2$$

$$\pi_2(S^2) = \mathbb{Z}$$

Topology in electronic systems



$$H(k) : S^1 \to S^1$$

$$\pi_1(S^1) = \mathbb{Z}$$

$$|\psi_n(\mathbf{r})\rangle = e^{i\mathbf{k}\cdot\mathbf{r}}|u_{n,\mathbf{k}}\rangle$$

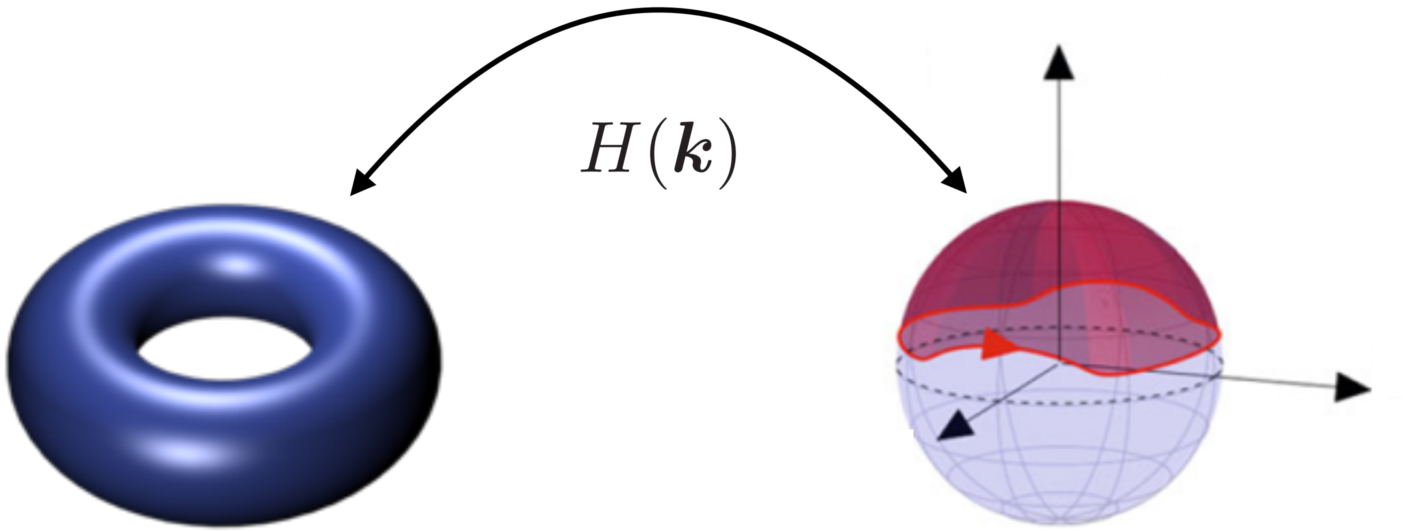
$$|u_{n,\mathbf{k}}\rangle$$

$$\theta_k = i\,\langle u_{n,k}|\,\partial_k\,|u_{n,k}\rangle$$

we have a phase defined on a circle in 1D

$$\phi_s = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk\partial_k\theta_k = \frac{1}{2\pi}\left(\theta_\pi - \theta_{-\pi}\right)$$

Topology in electronic systems



$$|\psi_n(\mathbf{r})\rangle = e^{i\mathbf{k}\cdot\mathbf{r}}|u_{n,\mathbf{k}}\rangle$$

$$|u_{n,\mathbf{k}}\rangle$$

$$\vec{\mathcal{A}}_n(\mathbf{k}) = i\langle u_{n,\mathbf{k}}|\vec{\nabla}_{\mathbf{k}}|u_{n,\mathbf{k}}\rangle$$

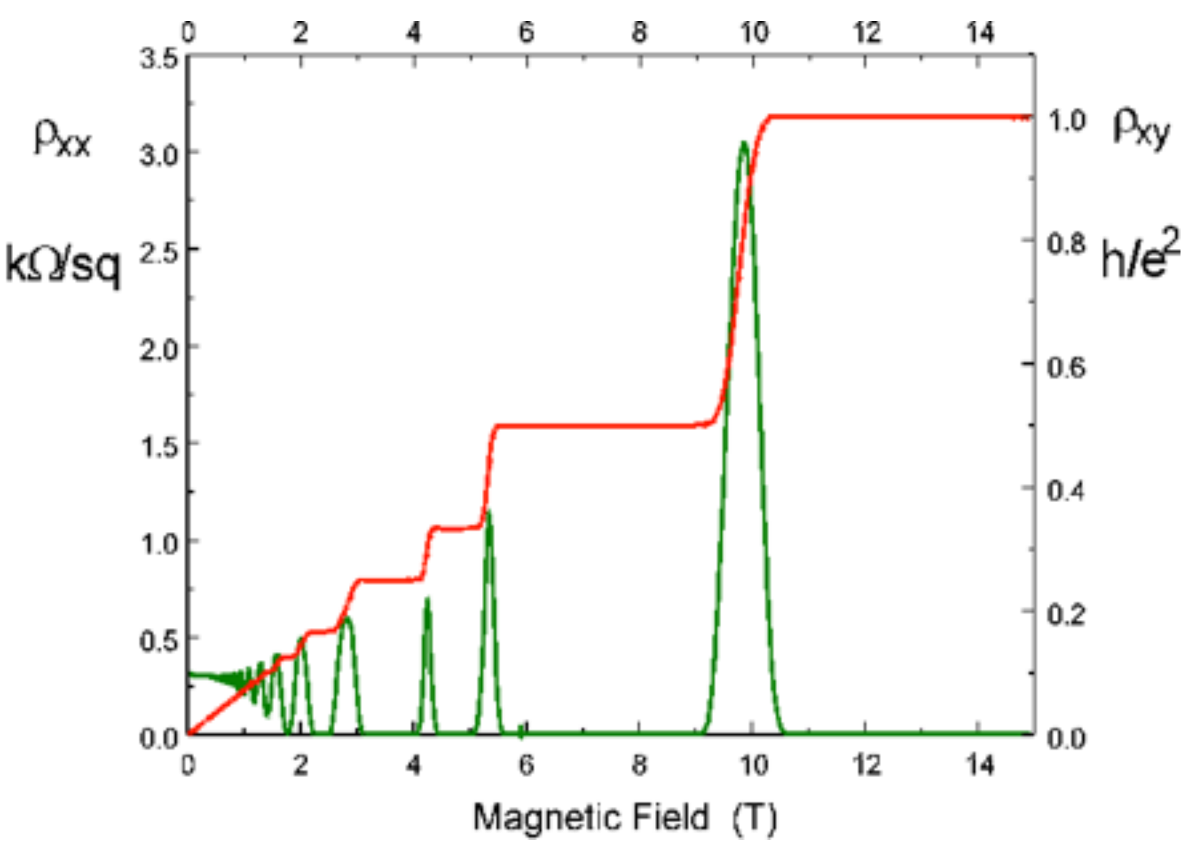
$$H(\boldsymbol{k}) : T^2 \to S^2$$

$$\pi_2(S^2) = \mathbb{Z}$$

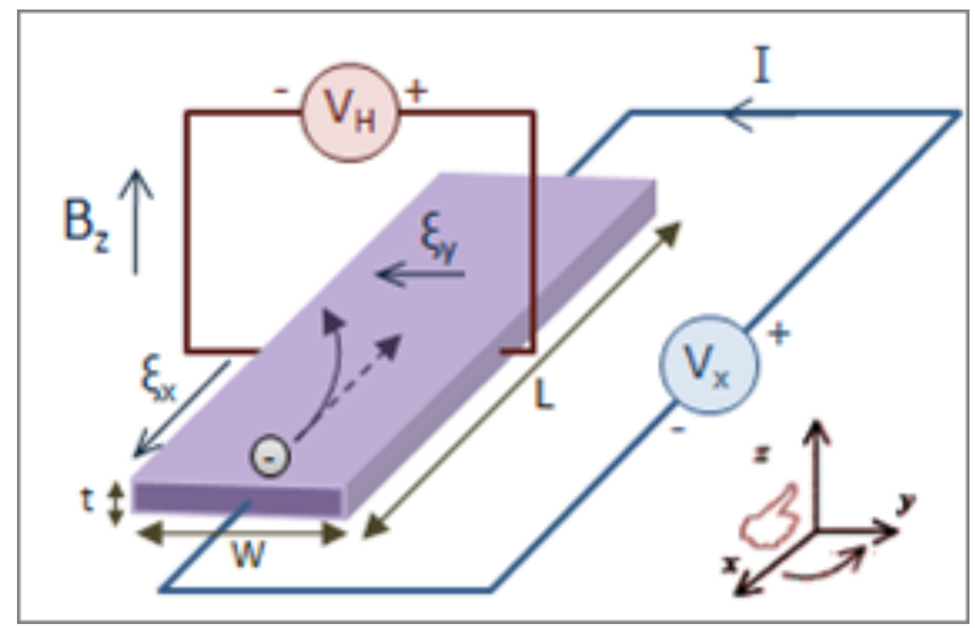
or a vector defined on a torus in 2D

$$\phi_n = \frac{1}{4\pi^2}\int_S d^2\boldsymbol{k}\cdot\partial_{\boldsymbol{k}}\times\boldsymbol{\mathcal{A}}_n$$

Topology in electronic systems



Quantum Hall effect



$$\sigma_H \equiv \frac{e^2}{2\pi\hbar} N$$

Topology in electronic systems

i) Hall conductance as integral of the Berry curvature

$$H = H(\boldsymbol{k}) + V \qquad V = e\boldsymbol{E}\cdot\boldsymbol{r} = -ie\boldsymbol{E}\cdot\partial_{\boldsymbol{k}}$$

$$H(\boldsymbol{k})\,|u_n(\boldsymbol{k})\rangle_0 = E_n(\boldsymbol{k})\,|u_n(\boldsymbol{k})\rangle_0 \qquad \boldsymbol{J} = e\boldsymbol{v} = e\frac{d\boldsymbol{r}}{dt} = \frac{e}{i\hbar}[H,\boldsymbol{r}] = \frac{e}{\hbar}\frac{\partial H}{\partial \boldsymbol{k}}$$

$$|\tilde{u}_n(\boldsymbol{k})\rangle = |u_n(\boldsymbol{k})\rangle - ieE_i\sum_{m\neq n}|u_m(\boldsymbol{k})\rangle\,\frac{\langle u_m(\boldsymbol{k})|\,\partial_{k_i}\,|u_n(\boldsymbol{k})\rangle}{E_m - E_n}$$

$$\langle\tilde{u}_n(\boldsymbol{k})| = \langle u_n(\boldsymbol{k})| + ieE_i\sum_{m\neq n}\frac{\langle u_n(\boldsymbol{k})|\,\partial_{k_i}\,|u_m(\boldsymbol{k})\rangle}{E_m - E_n}\,\langle u_m(\boldsymbol{k})|$$

first order perturbation theory!

$$\langle\boldsymbol{J}\rangle = \frac{1}{4\pi^2}\int d^2\boldsymbol{k}\,\langle\tilde{u}_n(\boldsymbol{k})|\,\boldsymbol{J}\,|\tilde{u}_n(\boldsymbol{k})\rangle$$

DJ Thouless, M Kohmoto, MP Nightingale, M Den Nijs, PRL, 49, 405 (1982)

Topology in electronic systems

i) Hall conductance as integral of the Berry curvature

$$\langle \boldsymbol{J} \rangle = \frac{1}{4\pi^2} \int d^2\boldsymbol{k} \, \langle \tilde{u}_n(\boldsymbol{k}) | \, \boldsymbol{J} \, | \tilde{u}_n(\boldsymbol{k}) \rangle$$

$$\langle J_j \rangle = i \frac{e^2}{4\pi^2} E_i \int d^2\boldsymbol{k} \sum_{m \neq n} \langle u_m(\boldsymbol{k}) | \, \partial_{k_j} \, | u_n(\boldsymbol{k}) \rangle \langle u_n(\boldsymbol{k}) | \, \partial_{k_i} \, | u_m(\boldsymbol{k}) \rangle -$$
$$- i \frac{e^2}{4\pi^2} E_i \int d^2\boldsymbol{k} \sum_{m \neq n} \langle u_n(\boldsymbol{k}) | \, \partial_{k_j} \, | u_m(\boldsymbol{k}) \rangle \langle u_m(\boldsymbol{k}) | \, \partial_{k_i} \, | u_n(\boldsymbol{k}) \rangle$$

$$\langle J_i \rangle = \frac{e^2}{2\pi h} E_j \int d^2\boldsymbol{k} \partial_{k_i} i \left\langle u_n(\boldsymbol{k}) | \partial_{k_j} u_n(\boldsymbol{k}) \right\rangle \qquad i \neq j$$

$$\sigma_{12} = \frac{e^2}{2\pi h} \int d^2\boldsymbol{k} \partial_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}(\boldsymbol{k}) \equiv \frac{e^2}{2\pi h} \int d^2\boldsymbol{k} \Omega(\boldsymbol{k})$$

Topology in electronic systems

i) Hall conductance as integral of the Berry curvature

$$\langle \boldsymbol{J} \rangle = \frac{1}{4\pi^2} \int d^2\boldsymbol{k} \, \langle \tilde{u}_n(\boldsymbol{k}) | \, \boldsymbol{J} \, | \tilde{u}_n(\boldsymbol{k}) \rangle$$

$$\langle J_j \rangle = i \frac{e^2}{4\pi^2} E_i \int d^2\boldsymbol{k} \sum_{m \neq n} \langle u_m(\boldsymbol{k}) | \, \partial_{k_j} \, | u_n(\boldsymbol{k}) \rangle \langle u_n(\boldsymbol{k}) | \, \partial_{k_i} \, | u_m(\boldsymbol{k}) \rangle -$$
$$- i \frac{e^2}{4\pi^2} E_i \int d^2\boldsymbol{k} \sum_{m \neq n} \langle u_n(\boldsymbol{k}) | \, \partial_{k_j} \, | u_m(\boldsymbol{k}) \rangle \langle u_m(\boldsymbol{k}) | \, \partial_{k_i} \, | u_n(\boldsymbol{k}) \rangle$$

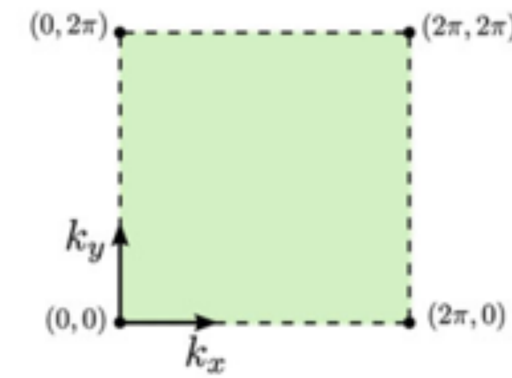
$$\langle J_i \rangle = \frac{e^2}{2\pi h} E_j \int d^2\boldsymbol{k} \partial_{k_i} i \left\langle u_n(\boldsymbol{k}) | \partial_{k_j} u_n(\boldsymbol{k}) \right\rangle \qquad i \neq j$$

$$\sigma_{12} = \frac{e^2}{2\pi h} \int d^2\boldsymbol{k} \partial_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}(\boldsymbol{k}) \equiv \frac{e^2}{2\pi h} \int d^2\boldsymbol{k} \Omega(\boldsymbol{k})$$

Topology in electronic systems

ii) Quantization of the Hall conductance

$$\sigma_{12} = \frac{e^2}{2\pi h}\int d^2\boldsymbol{k}\,\partial_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}(\boldsymbol{k}) \qquad \mathcal{A}_i = i\,\langle u(k_1,k_2)|\,\partial_{k_i}\,|u(k_1,k_2)\rangle$$

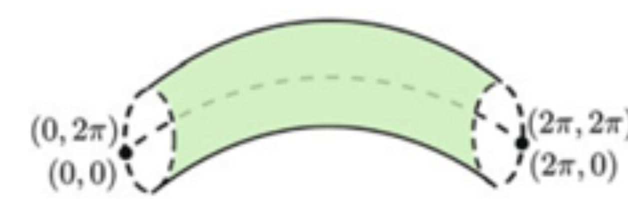


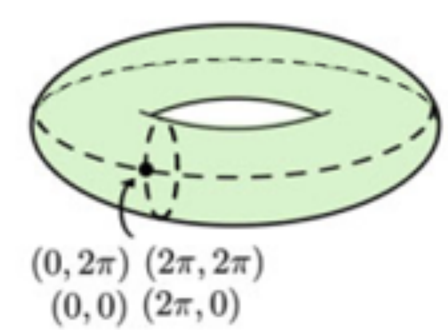
using Stokes theorem

$$\sigma_{12} = \frac{e^2}{2\pi h}\oint d\boldsymbol{k}\cdot\boldsymbol{\mathcal{A}}(\boldsymbol{k})$$

$$\frac{2\pi h}{e^2}\sigma_{12} = \int_0^{2\pi} dk_1 \mathcal{A}_1(k_1,0) - \int_0^{2\pi} dk_1 \mathcal{A}_1(k_1,2\pi) +$$

$$+\int_0^{2\pi} dk_2 \mathcal{A}_2(2\pi,k_2) - \int_0^{2\pi} dk_2 \mathcal{A}_2(0,k_2)$$

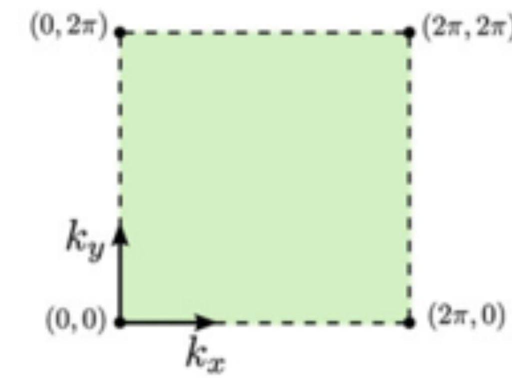




Topology in electronic systems

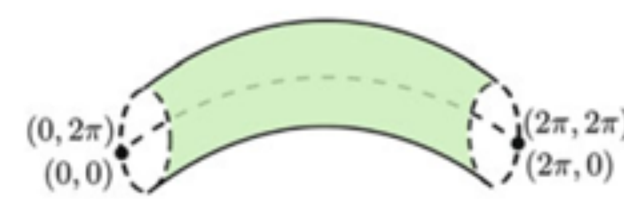
ii) Quantization of the Hall conductance

$\mathcal{A}_i = i \langle u(k_1, k_2) | \partial_{k_i} | u(k_1, k_2) \rangle$



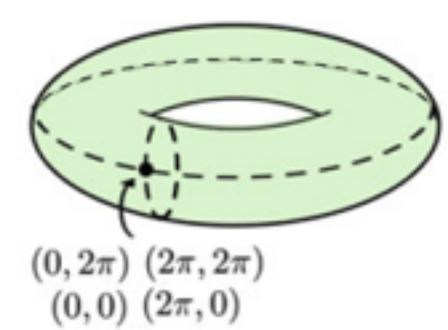
$$|u(k_1, 2\pi)\rangle = e^{i\theta_1(k_1)} |u(k_1, 0)\rangle$$

$$|u(2\pi, k_2)\rangle = e^{i\theta_2(k_2)} |u(0, k_2)\rangle$$



$$\mathcal{A}_1(k_1, 2\pi) = \mathcal{A}_1(k_1, 0) - \partial_1\theta_1(k_1)$$

$$\mathcal{A}_2(2\pi, k_2) = \mathcal{A}_2(0, k_2) - \partial_2\theta_2(k_2)$$

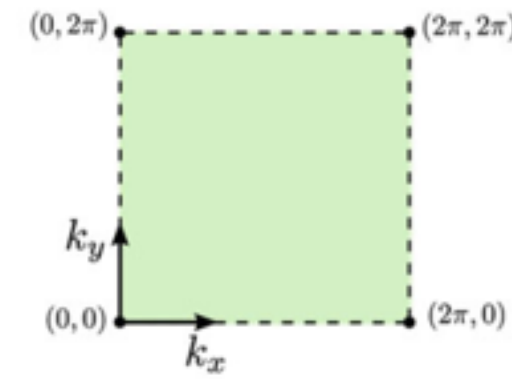


$$\frac{2\pi h}{e^2}\sigma_{12} = \int_0^{2\pi} dk_1 \partial_1\theta_1 - \int_0^{2\pi} dk_2 \partial_2\theta_2$$

Topology in electronic systems

ii) Quantization of the Hall conductance

$\mathcal{A}_i = i \langle u(k_1, k_2)| \partial_{k_i} |u(k_1, k_2)\rangle$



$$|u(k_1, 2\pi)\rangle = e^{i\theta_1(k_1)} |u(k_1, 0)\rangle$$

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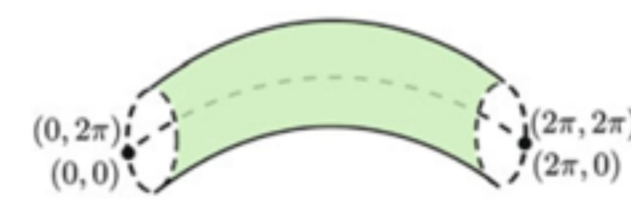
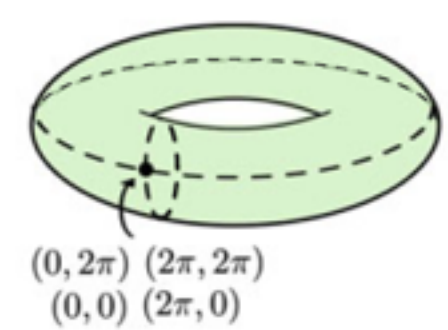


$$\mathcal{A}_1(k_1, 2\pi) = \mathcal{A}_1(k_1, 0) - \partial_1\theta_1(k_1)$$

$$\mathcal{A}_2(2\pi, k_2) = \mathcal{A}_2(0, k_2) - \partial_2\theta_2(k_2)$$

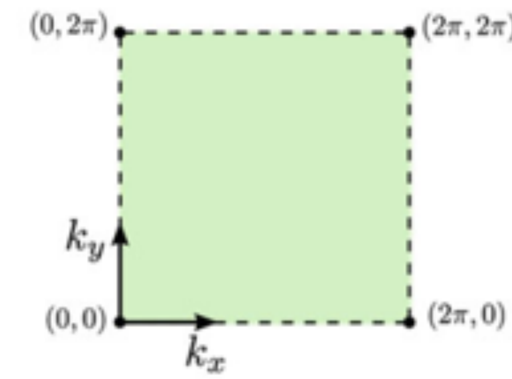


$$\frac{2\pi h}{e^2}\sigma_{12} = \theta_1(2\pi) - \theta_1(0) - \theta_2(2\pi) + \theta_2(0)$$

Topology in electronic systems

ii) Quantization of the Hall conductance

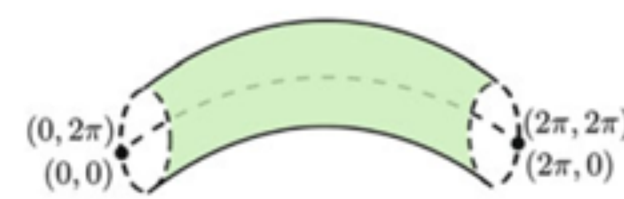
$$\frac{2\pi h}{e^2}\sigma_{12} = \theta_1(2\pi) - \theta_1(0) - \theta_2(2\pi) + \theta_2(0)$$



$$|u(2\pi, k_2)\rangle = e^{i\theta_2(k_2)}\,|u(0, k_2)\rangle$$

$$|u(2\pi, 0)\rangle = e^{i\theta_2(0)}\,|u(0, 0)\rangle$$

$$|u(2\pi, 2\pi)\rangle = e^{i\theta_2(2\pi)}\,|u(0, 2\pi)\rangle$$

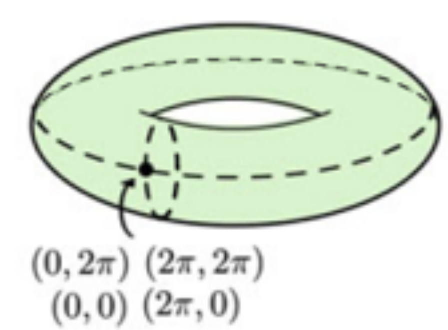


$$|u(k_1, 2\pi)\rangle = e^{i\theta_1(k_1)}\,|u(k_1, 0)\rangle$$

$$|u(0, 2\pi)\rangle = e^{i\theta_1(0)}\,|u(0, 0)\rangle$$

$$|u(2\pi, 2\pi)\rangle = e^{i\theta_1(2\pi)}\,|u(2\pi, 0)\rangle$$

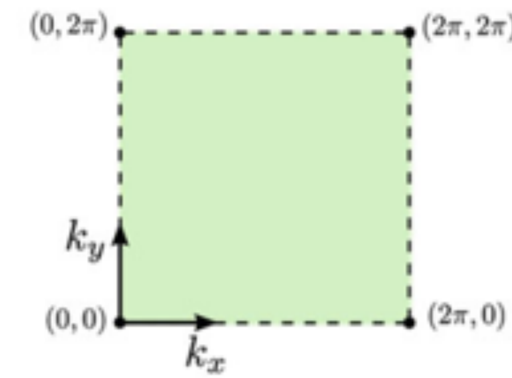
$$|u(2\pi, 2\pi)\rangle = e^{i(\theta_1(2\pi)+\theta_2(0)-\theta_1(0)-\theta_2(2\pi))}\,|u(2\pi, 2\pi)\rangle$$



Topology in electronic systems

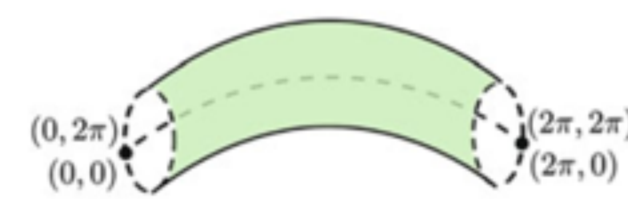
ii) Quantization of the Hall conductance

$$\frac{2\pi h}{e^2}\sigma_{12} = \theta_1(2\pi) - \theta_1(0) - \theta_2(2\pi) + \theta_2(0)$$



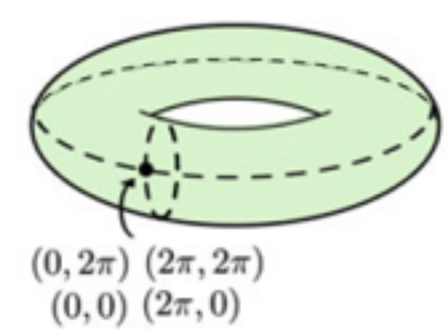
$$|u(2\pi, 2\pi)\rangle = e^{i(\theta_1(2\pi)+\theta_2(0)-\theta_1(0)-\theta_2(2\pi))}\,|u(2\pi, 2\pi)\rangle$$

$$|u(2\pi, 2\pi)\rangle = e^{2\pi N i}\,|u(2\pi, 2\pi)\rangle$$



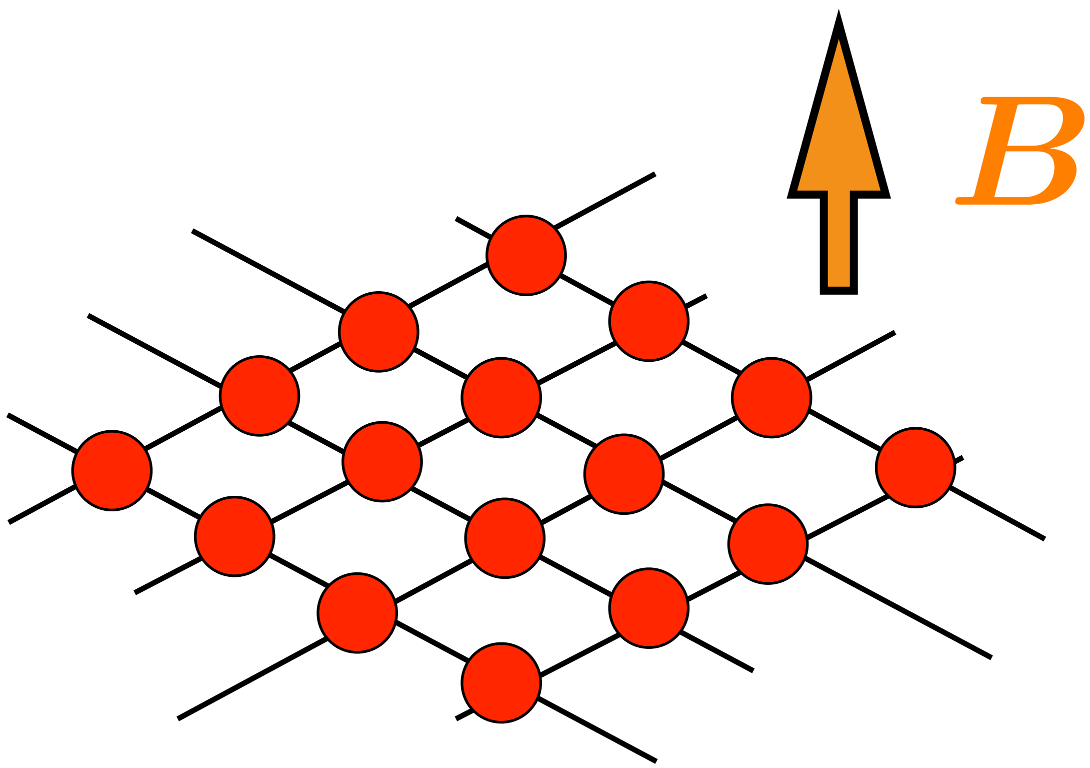
$$\theta_1(2\pi) + \theta_2(0) - \theta_1(0) - \theta_2(2\pi) = 2\pi N$$

$$\sigma_{12} = \frac{e^2}{h}N$$



Topology in electronic systems

Presence of a magnetic field



Landau levels

$$t_{ij} \to t_{ij} e^{i \int_i^j \boldsymbol{A} \cdot d\boldsymbol{r}}$$

$$(x, y) = a(n, m) \qquad \boldsymbol{A} = B(am, 0, 0)$$

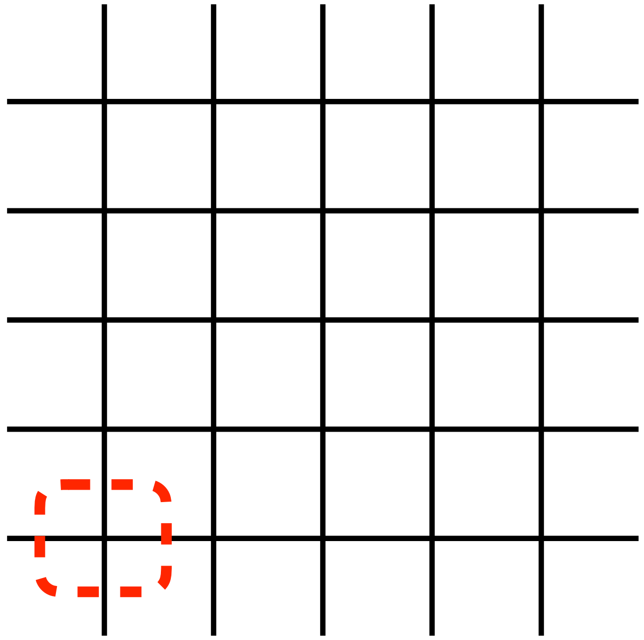
$$t_{n,n\pm 1} \to t e^{\pm i B a^2 m}$$

$$m \to m + \frac{2\pi N}{Ba^2}$$

we recover "lattice" periodicity

Topology in electronic systems

Presence of a magnetic field



we recover
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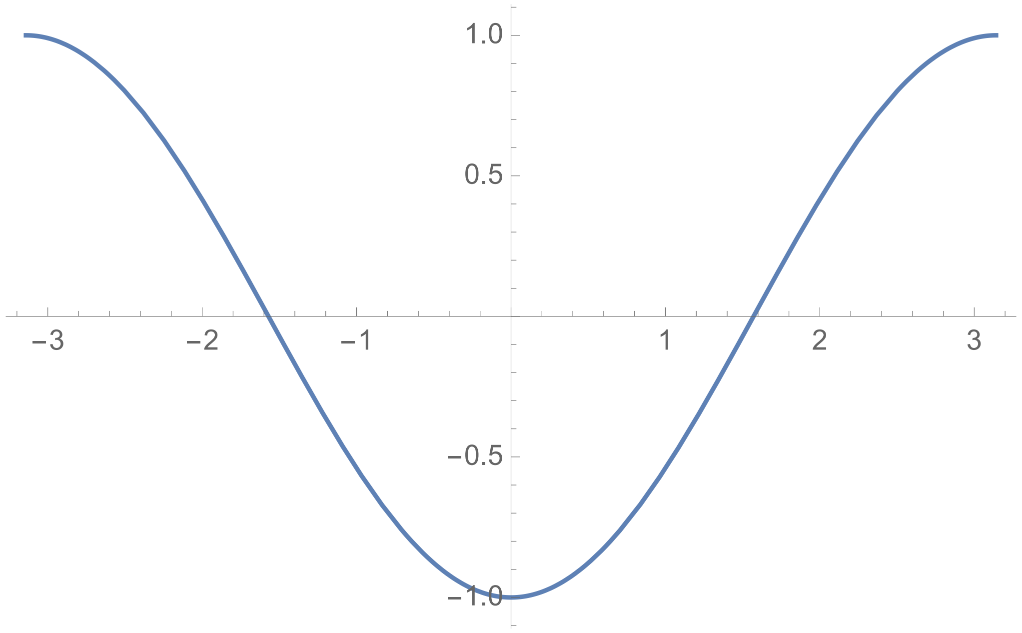
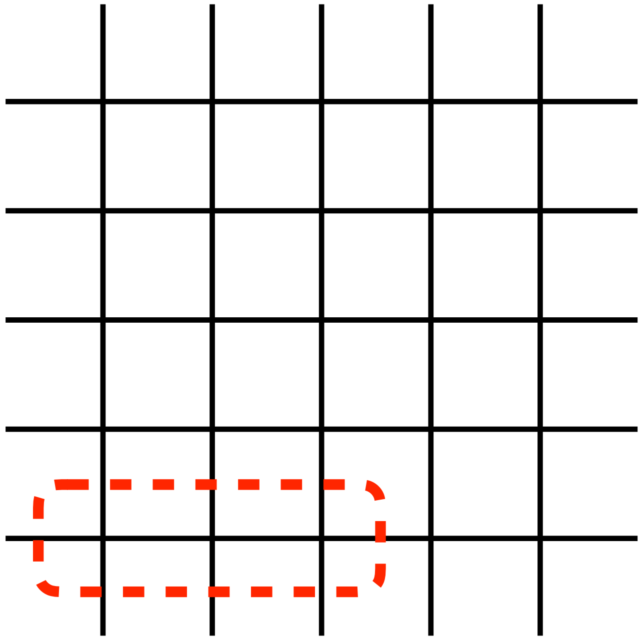


Topology in electronic systems

Presence of a magnetic field



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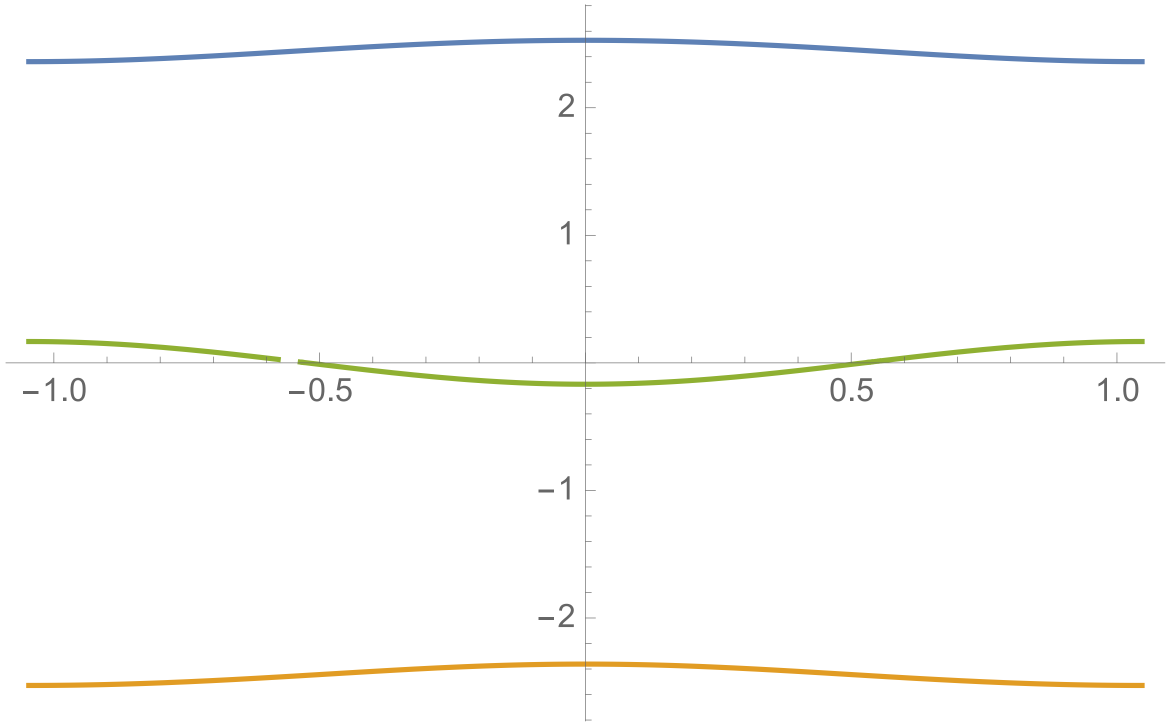

Topology in electronic systems

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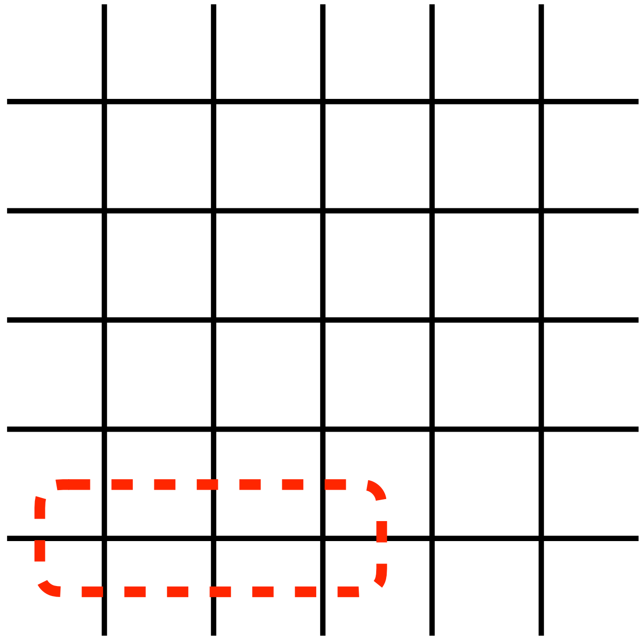

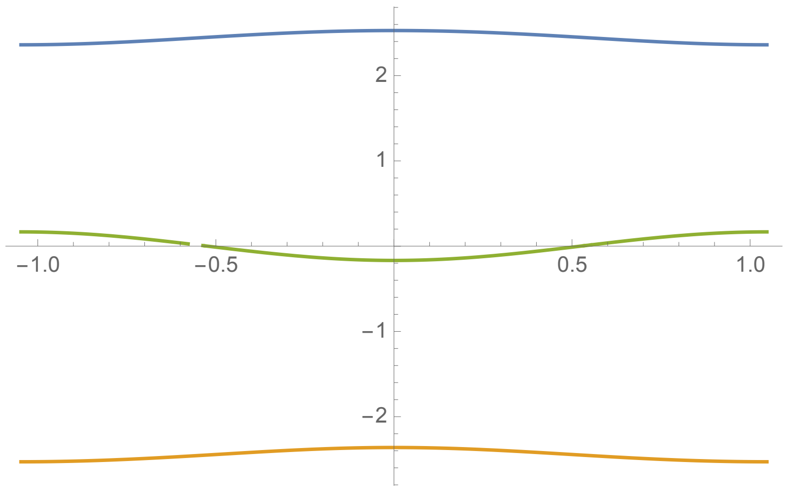

$$\mathcal{A}_i = i \langle u(k_1, k_2) | \partial_{k_i} | u(k_1, k_2) \rangle$$

$$\sigma_{12} = \frac{e^2}{2\pi h} \int d^2\boldsymbol{k} \partial_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}(\boldsymbol{k})$$

Topology in electronic systems

Presence of a magnetic field

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

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F. D. M. Haldane

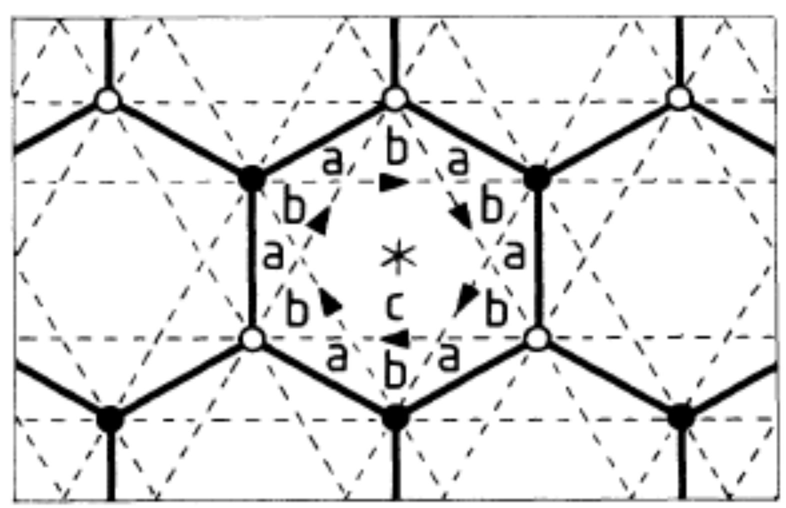
Department of Physics, University of California, San Diego, La Jolla, California 92093

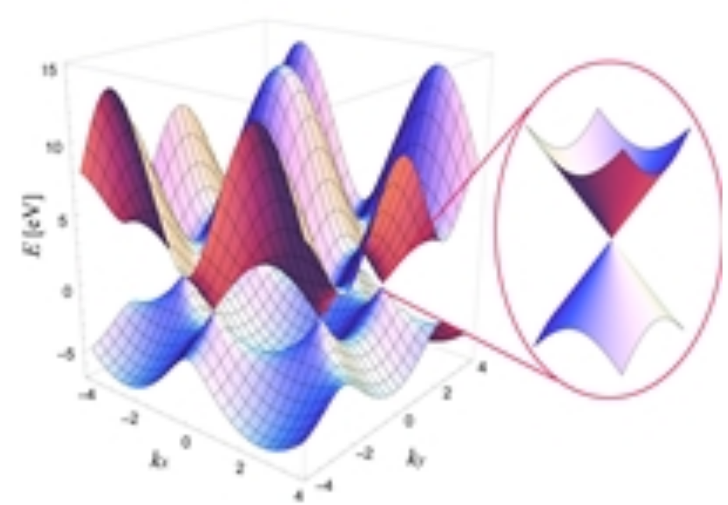
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.

$$t_{ij} \to t_{ij} e^{i \int_i^j \boldsymbol{A} \cdot d\boldsymbol{r}}$$

$$te^{i\theta}$$





Topology in electronic systems

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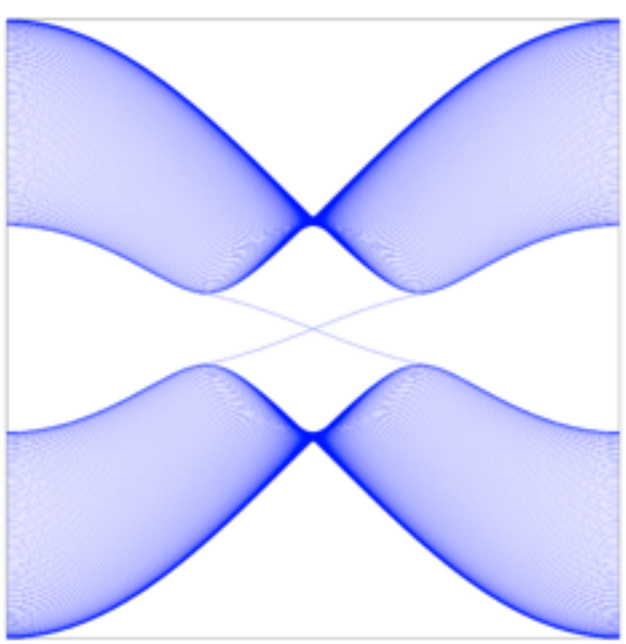

$$\mathcal{S} = \int d^3x C \epsilon_{ili} A_i \partial_l A_j - J_i A_i$$

$$J_i = C \epsilon_{ij} E_j$$

$$m \sim t_2 \sin\theta$$

~~$C \sim B$~~ $\qquad$ $C \sim sign(m)$

Topology in electronic systems

Presence of a magnetic field

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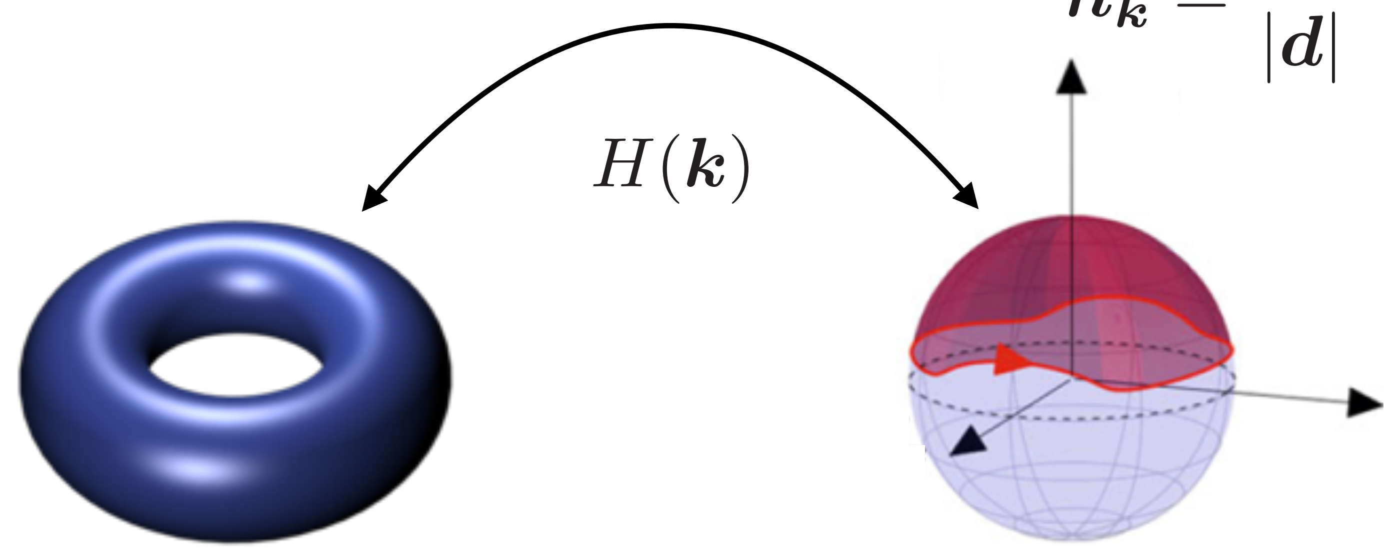
$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{d}(\boldsymbol{k})$$

$$C = \int d^2\boldsymbol{k}\boldsymbol{\partial} \times \boldsymbol{\mathcal{A}} = \int d^2\boldsymbol{k}\boldsymbol{n} \cdot \partial_1 \boldsymbol{n} \times \partial_2 \boldsymbol{n}$$

$$\boldsymbol{n}_{\boldsymbol{k}} = \frac{\boldsymbol{d}}{|\boldsymbol{d}|}$$

$$C \sim sign(m)$$

$$m \sim t_2 \sin\theta$$



Topology in electronic systems

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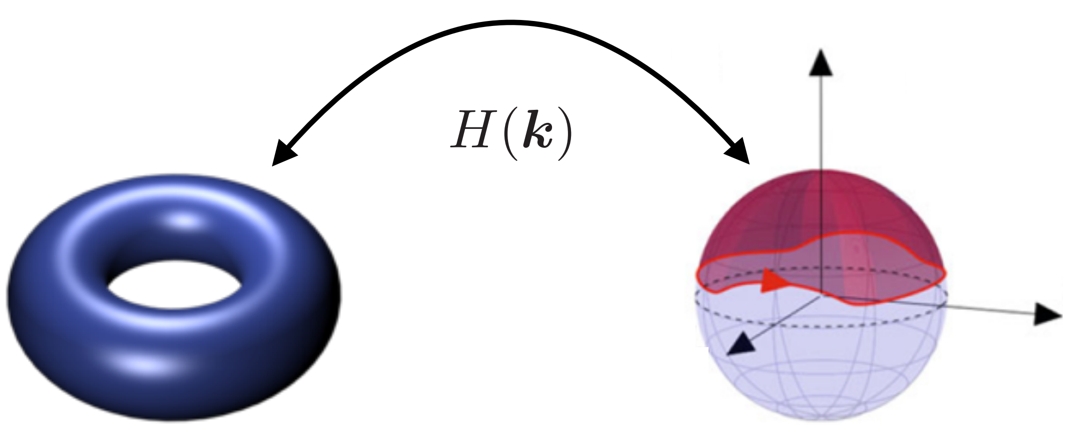
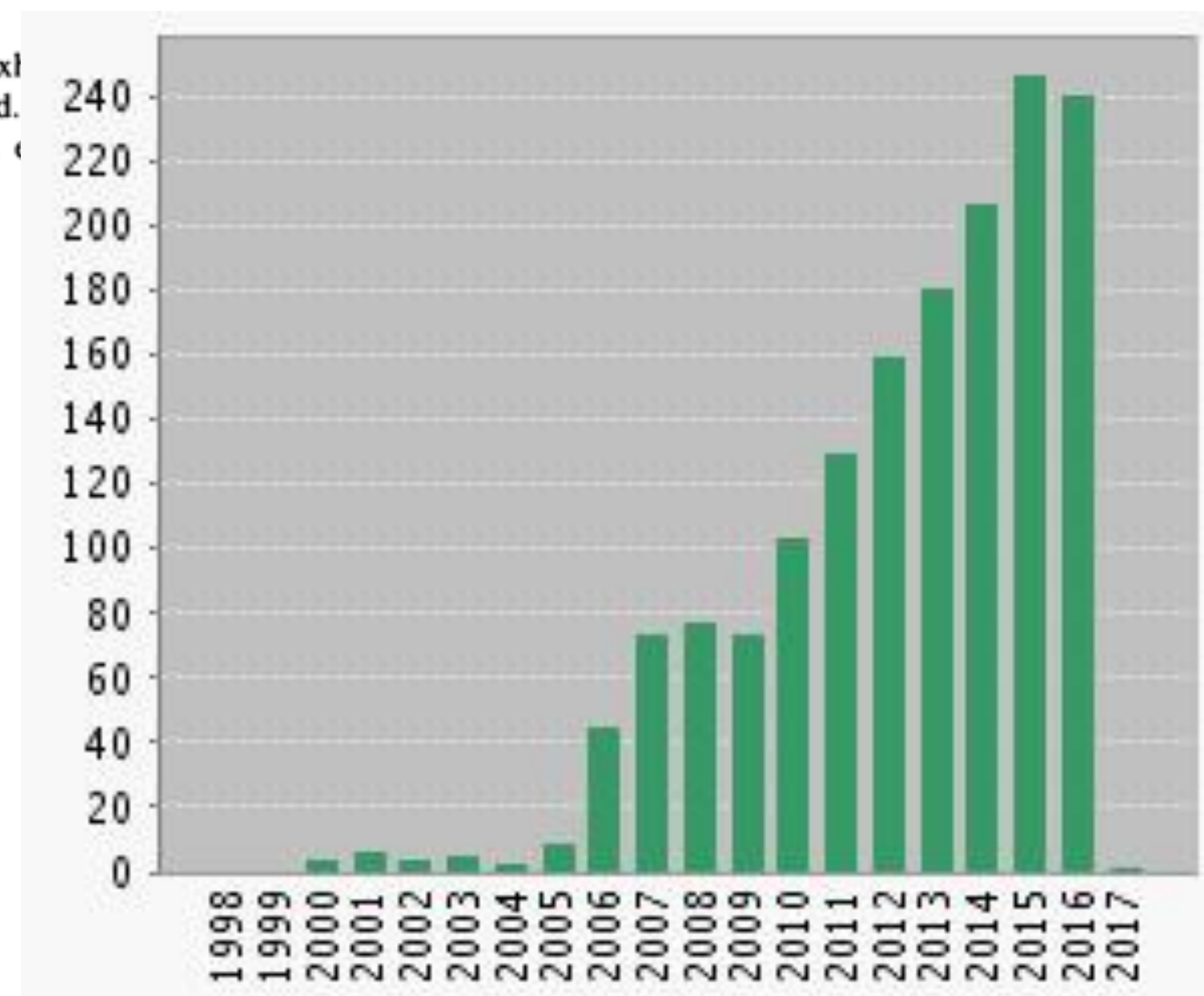
$$C \sim sign(m)$$

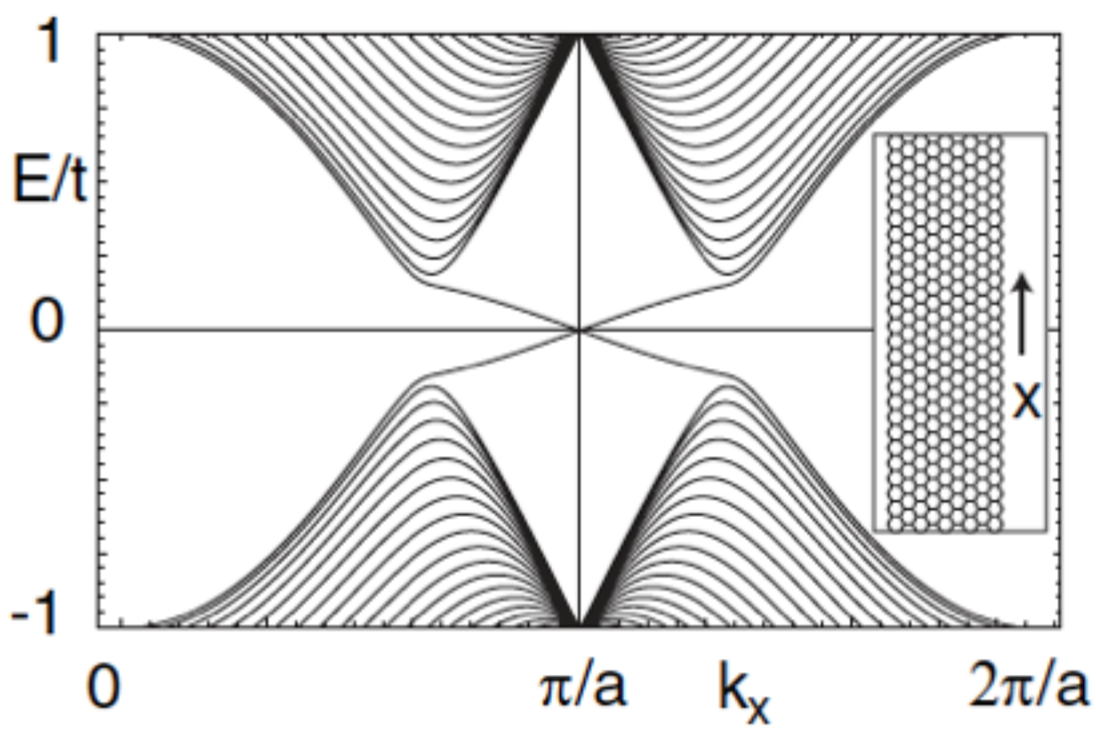




Topology in electronic systems

graphene + spin-orbit

$\times 2$



time reversal invariant

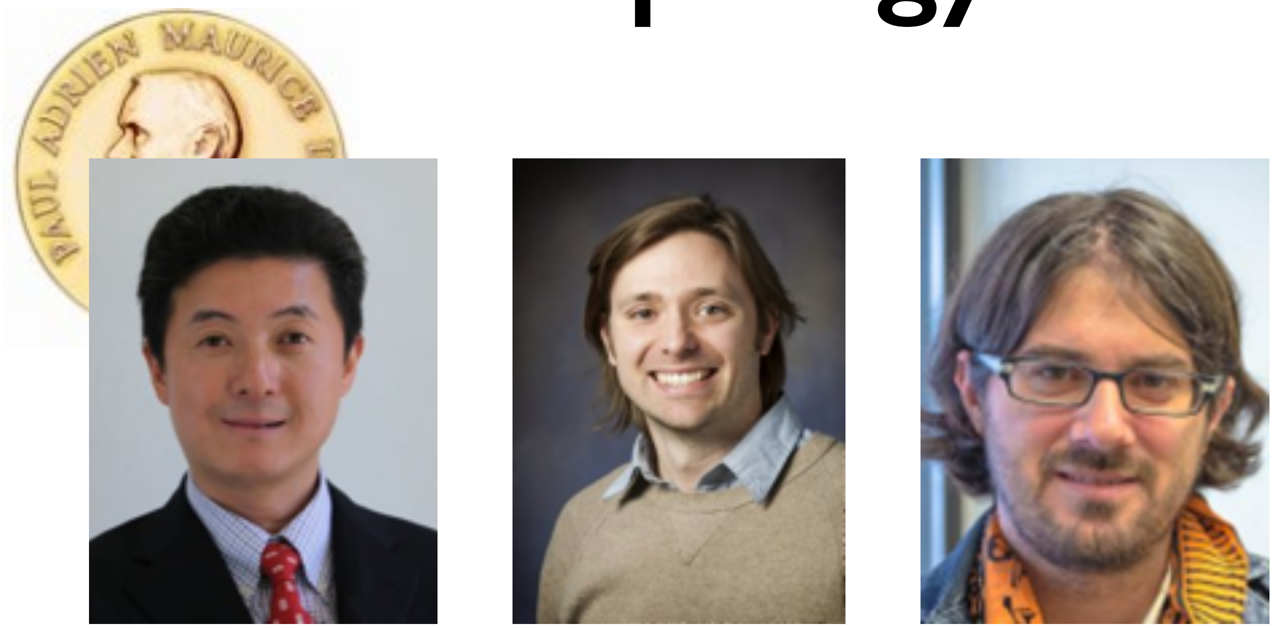
$$\sigma_{12} = 0$$

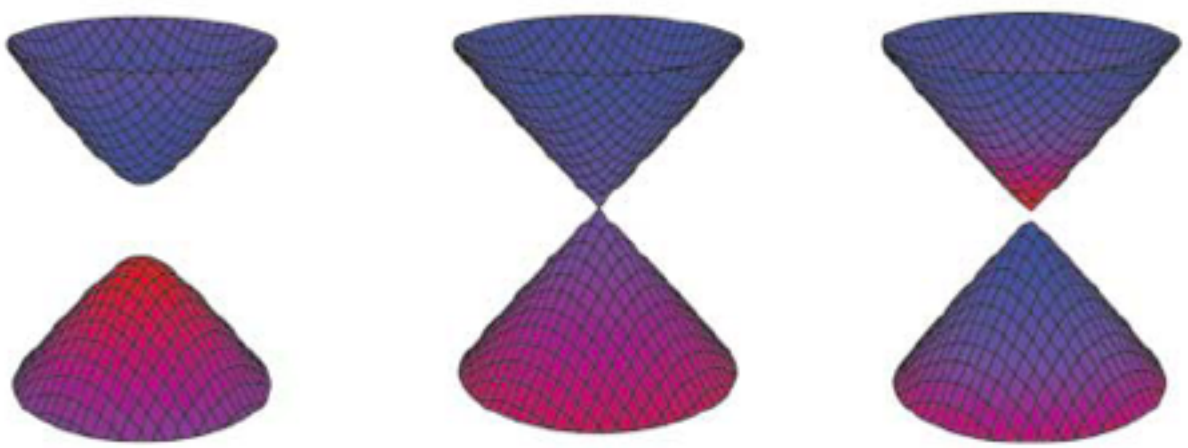
$$H_{SO} = \lambda \boldsymbol{L} \cdot \boldsymbol{S}$$

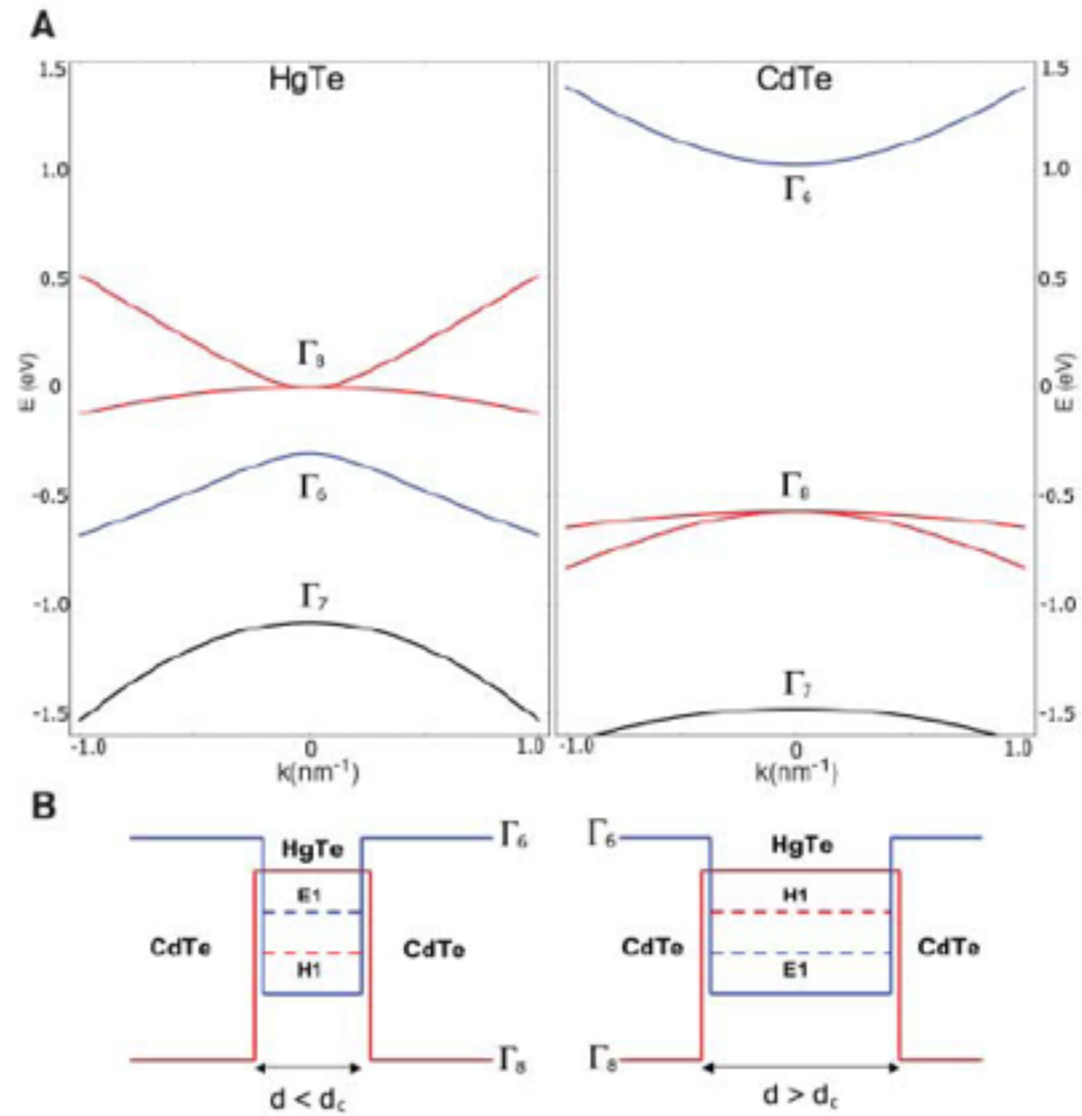
$$\sigma_{12}^{\uparrow} \neq 0 \qquad \sigma_{12}^{\uparrow} = -\sigma_{12}^{\downarrow}$$

CL Kane, E Mele, PRL, 95, 226801 (2005)

Topology in electronic systems







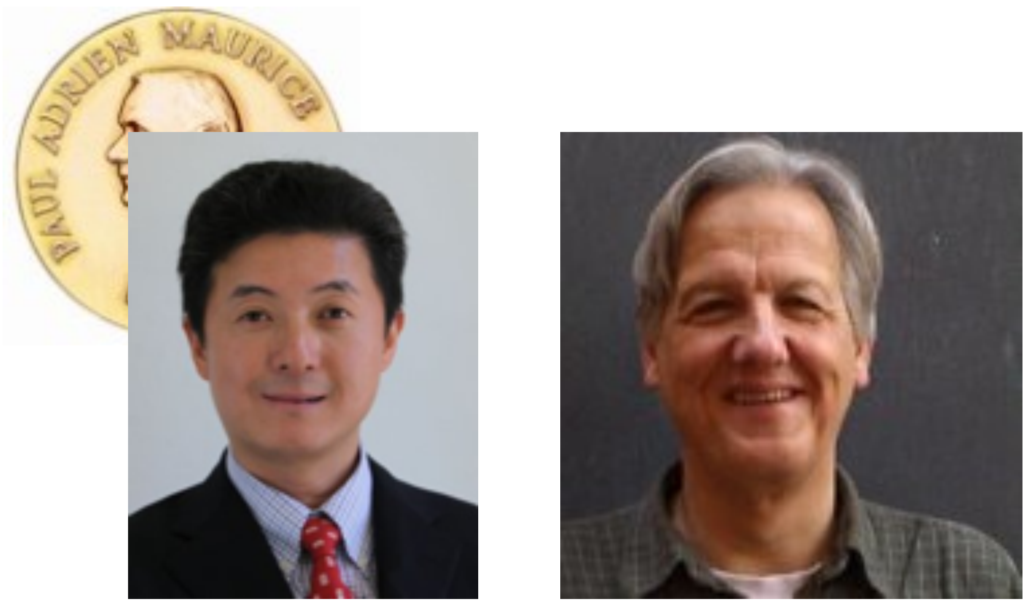
$$H(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{d}(\boldsymbol{k})$$

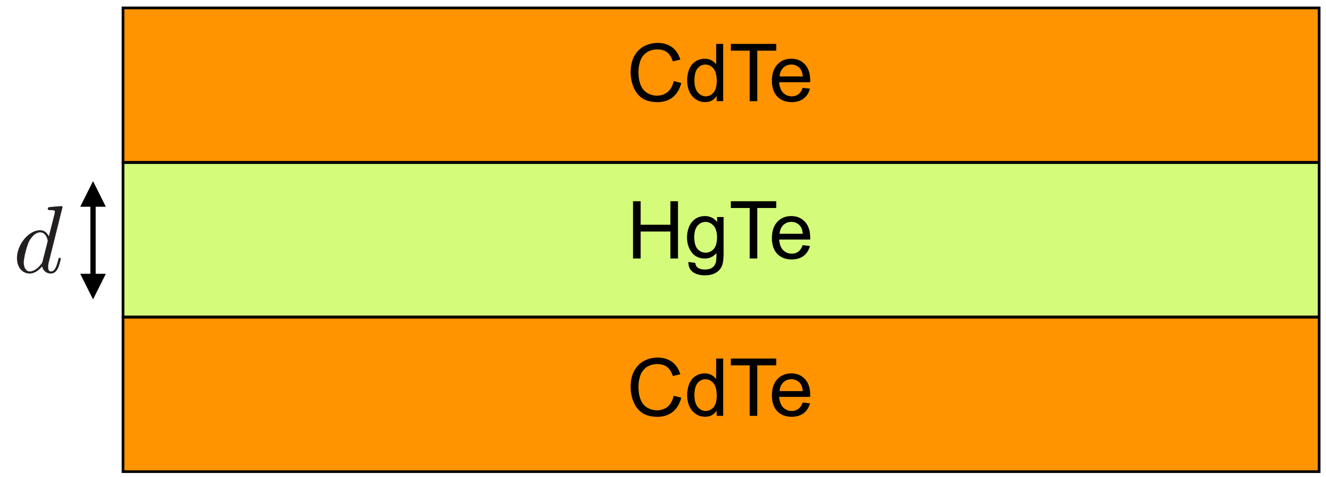
$$C = \int d^2\boldsymbol{k} \epsilon_{abc} n_a (\partial_1 n_b \partial_2 n_c)$$

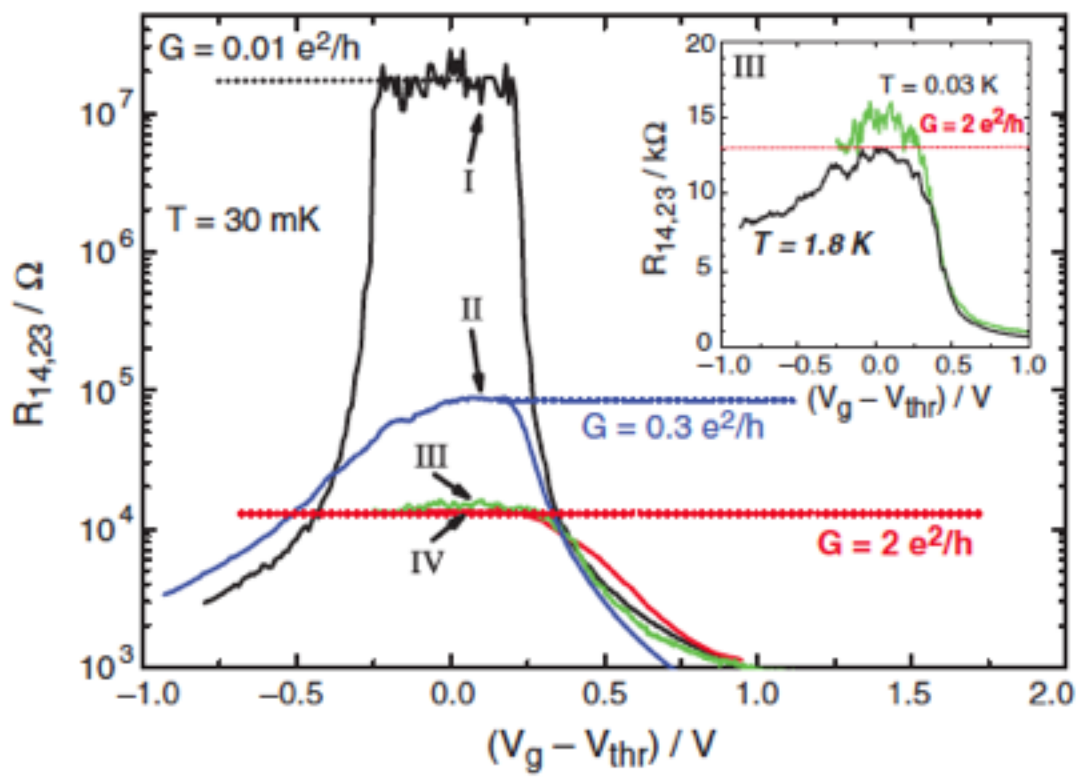
$$\sigma_{12}^{\uparrow} = -\sigma_{12}^{\downarrow}$$

BA Bernevig, TL Hughes, SC Zhang, Science, 314, 757 (2006)

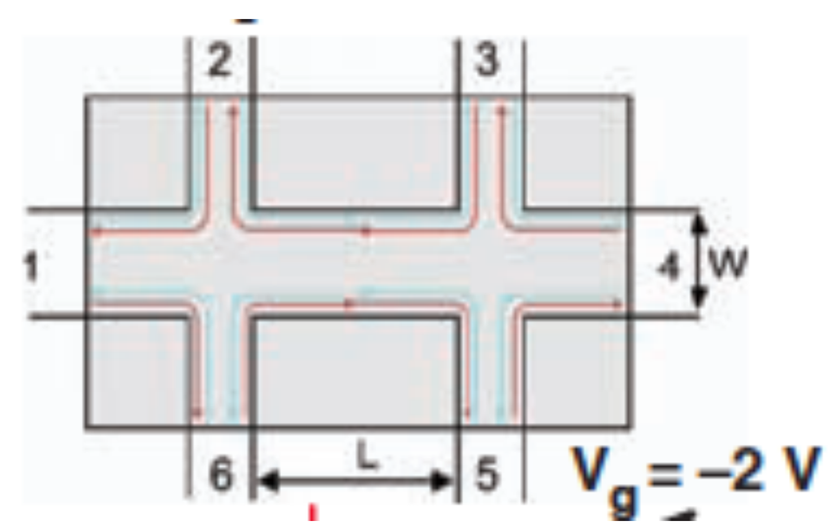
Topology in electronic systems





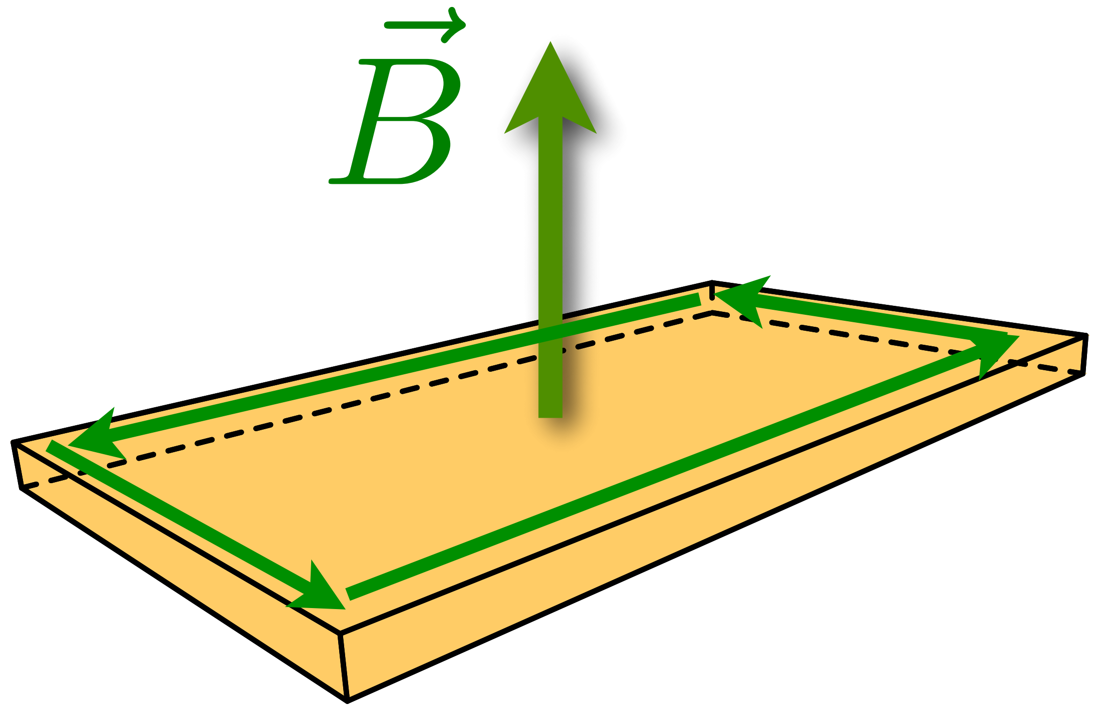


M König et al. Science, 38, 766 (2007)



conduction through the edges

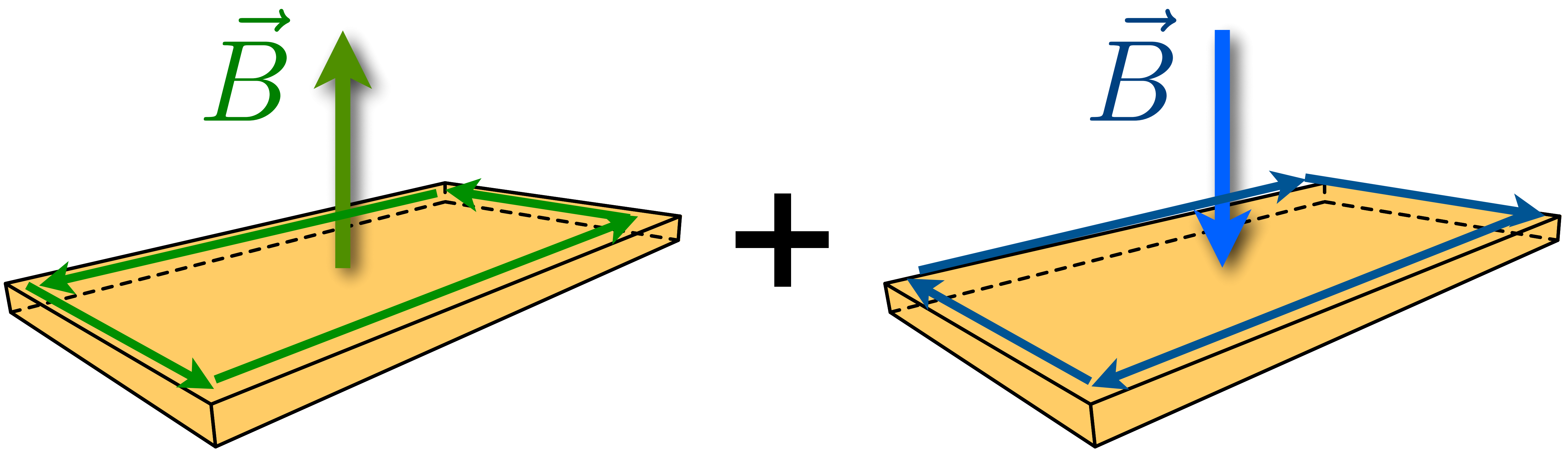
Topology in electronic systems



Topology in electronic systems

Time reversal symmetry would force

$$t \to -t$$



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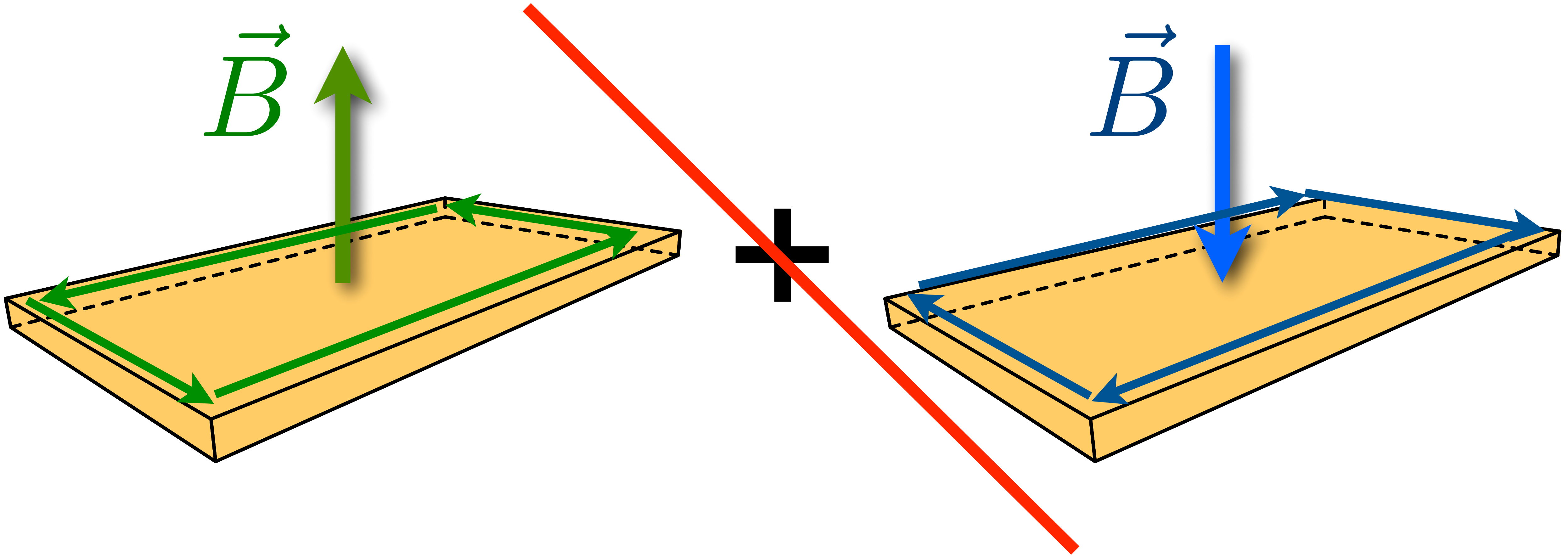

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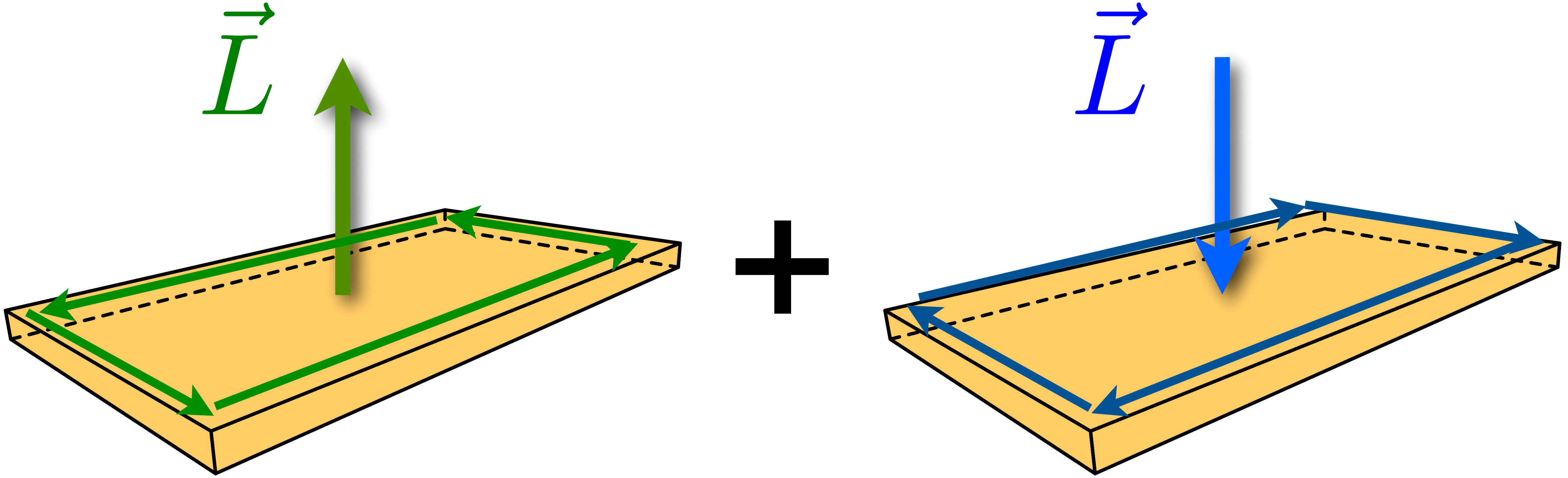

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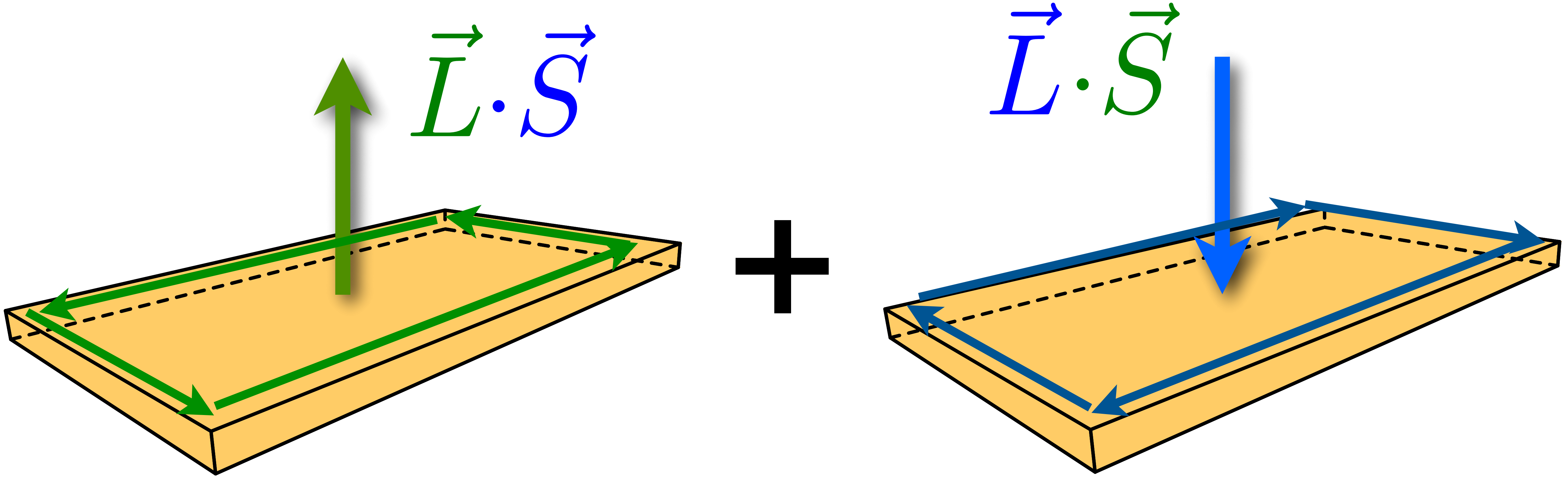
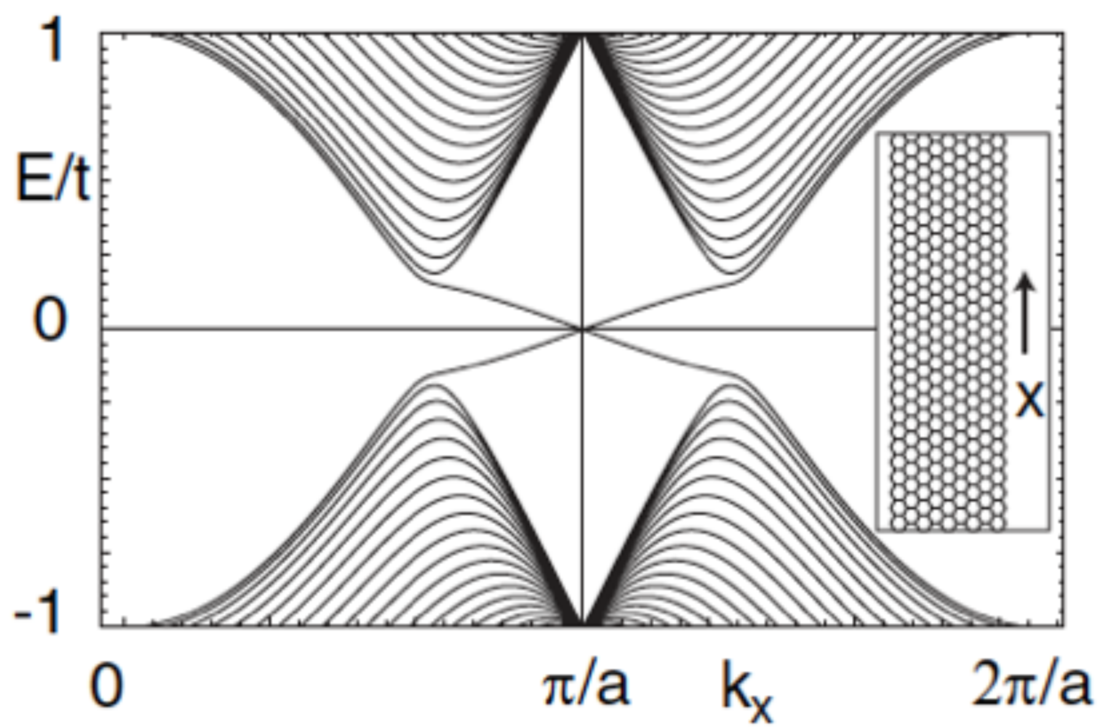


$$H_{SO} = \lambda \boldsymbol{L} \cdot \boldsymbol{S}$$

Topology in electronic systems

Edge states Bulk-boundary correspondence



Topology in electronic systems

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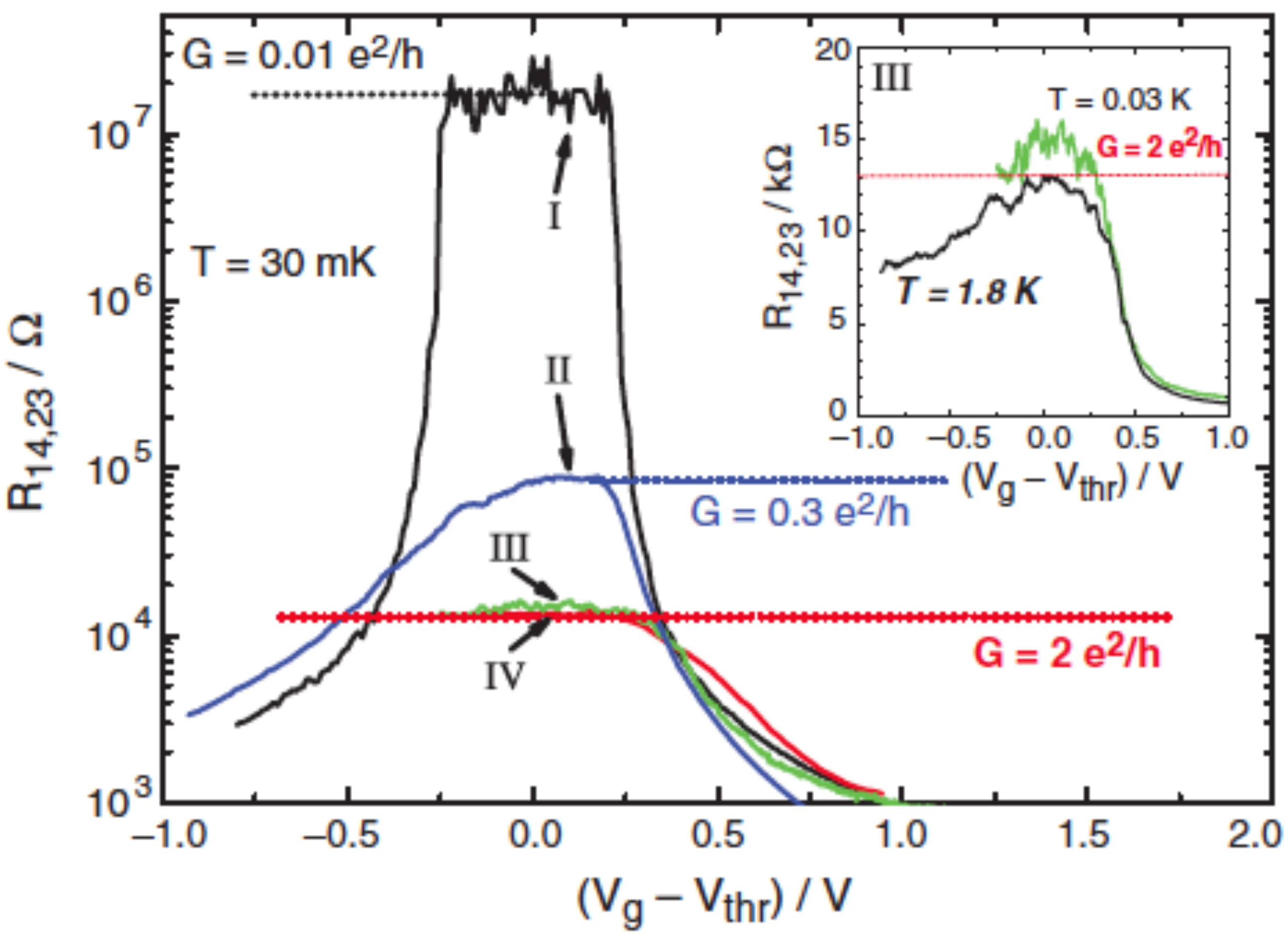
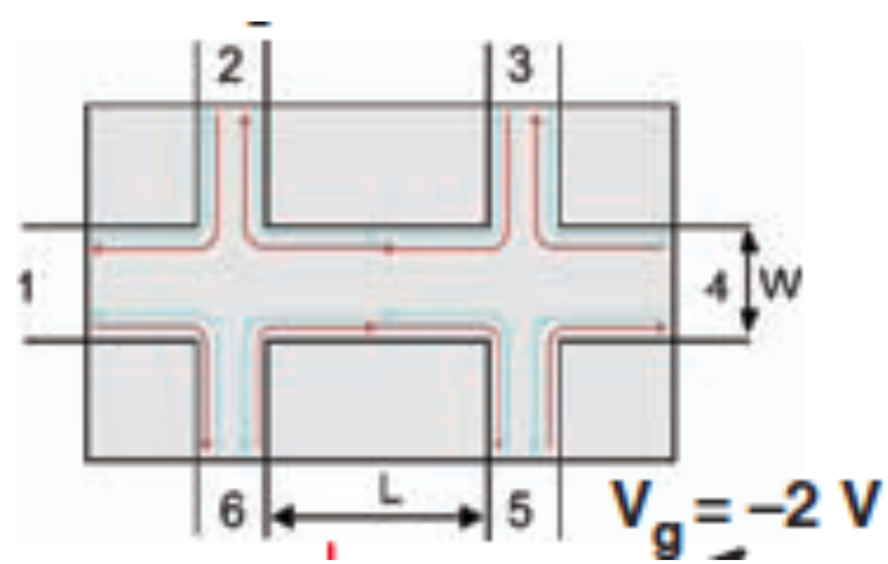
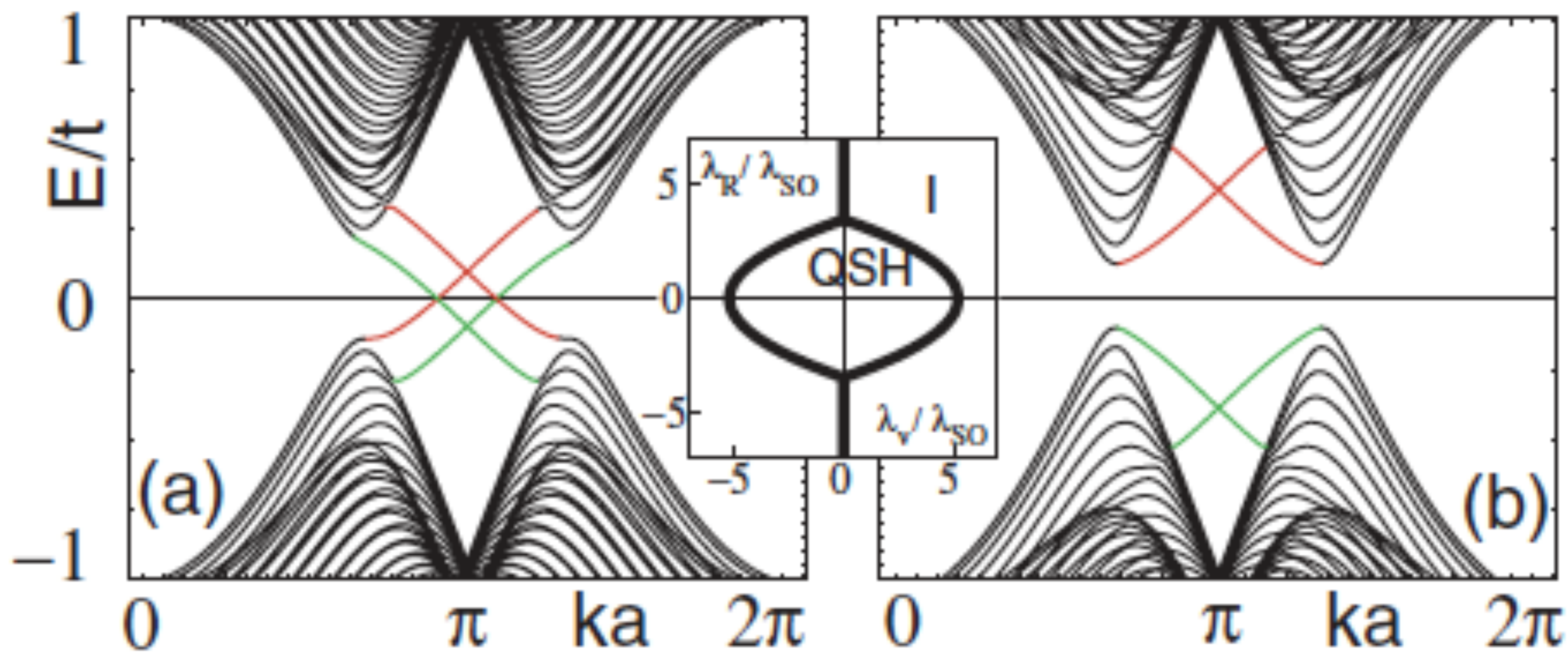




Topology in electronic systems

Edge states Bulk-boundary correspondence



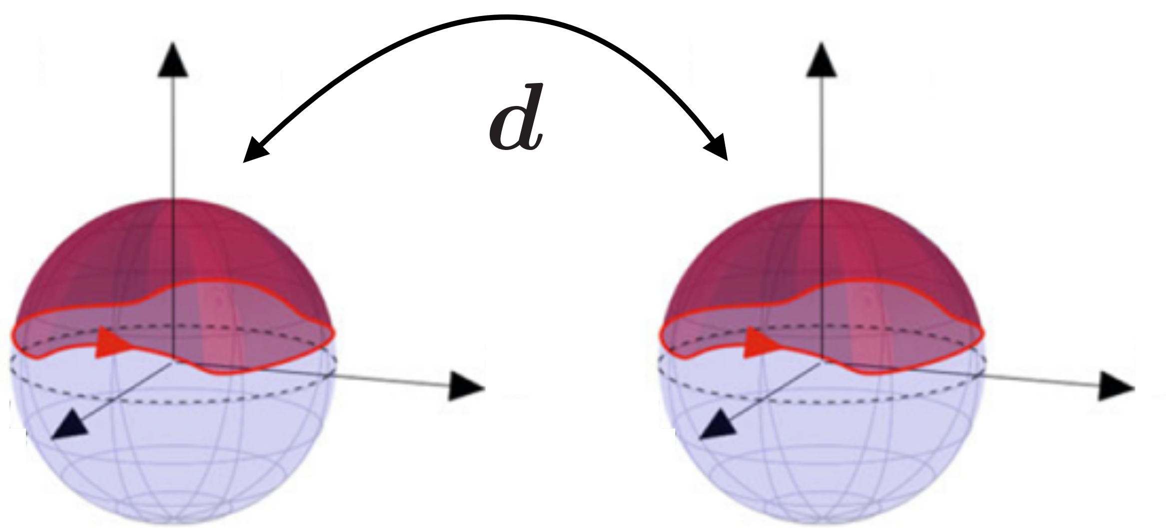
Modern notion of
symmetry protected topological phases

Topology in electronic systems

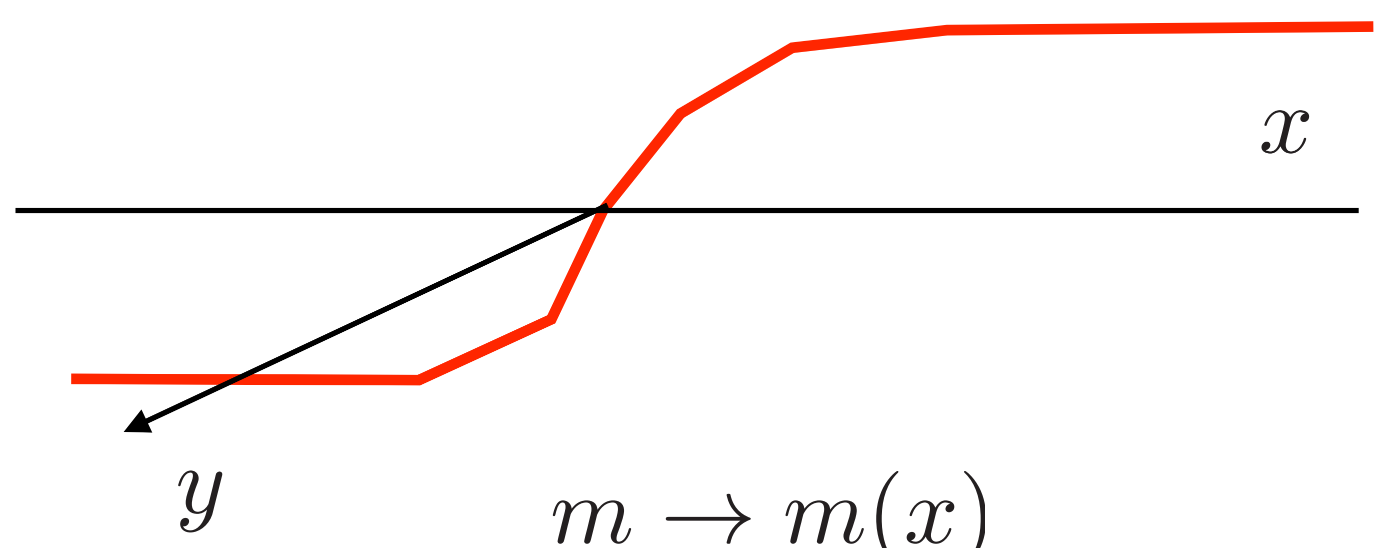
Edge states Bulk-boundary correspondence

Dirac equation in the continuum

$$\mathcal{Q} = \frac{1}{8\pi^3} \int d^2\boldsymbol{k} \epsilon_{abc} n_a \partial_1 n_b \partial_2 n_c$$



$$\mathcal{Q} = \frac{1}{2} \frac{m}{|m|}$$

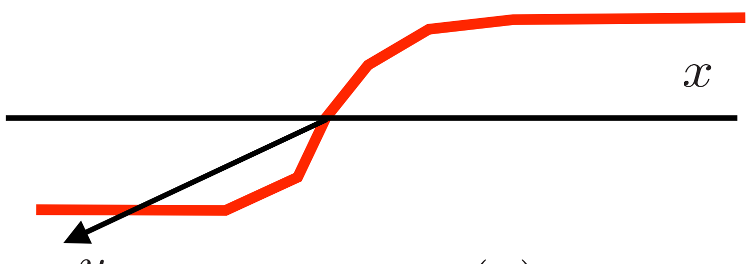


$$H = \boldsymbol{\sigma} \cdot \boldsymbol{d} = \sigma_1 k_1 + \sigma_2 k_2 + \sigma_3 m$$

$$m \to m(x)$$

Topology in electronic systems

Edge states Bulk-boundary correspondence

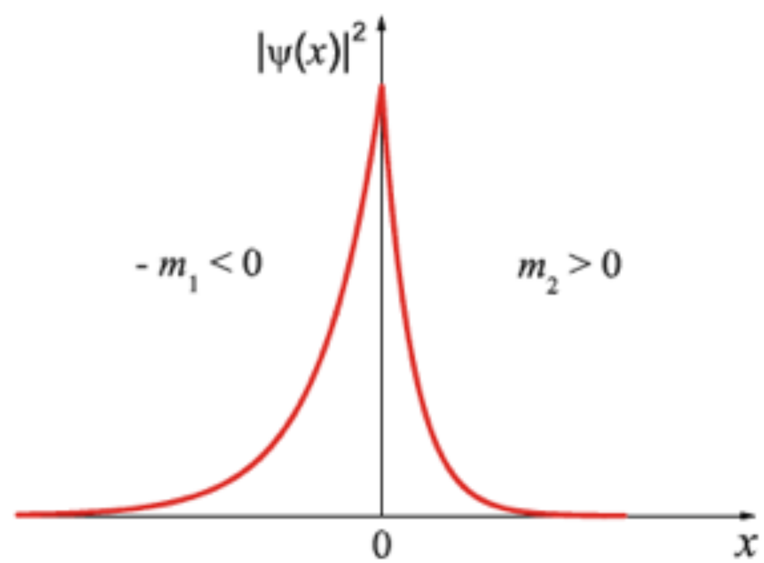


$$m \to m(x)$$

$$H = -i\sigma_1 \partial_1 + \sigma_3 m(x) + \sigma_2 k$$

$$E = 0$$

$$\psi = e^{iky} e^{-\int_x m(z)dz} a \left| s \right\rangle$$

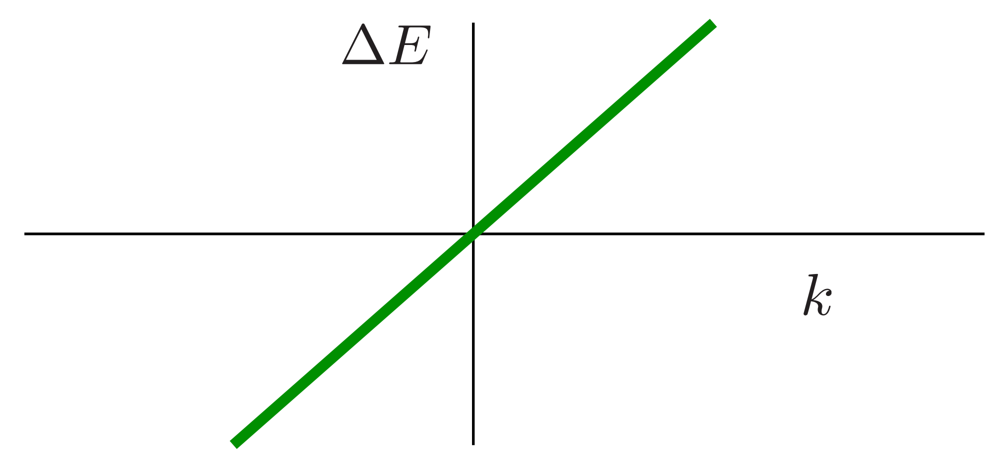


$$E_{\boldsymbol{k}} = \pm\sqrt{\boldsymbol{k}^2 + m^2}$$

$$\psi_{\boldsymbol{k}}^{\alpha} = a_1^{\alpha} \left| 1 \right\rangle + a_{-1}^{\alpha} \left| -1 \right\rangle$$

$$\sigma_r \left| s \right\rangle = s \left| s \right\rangle$$

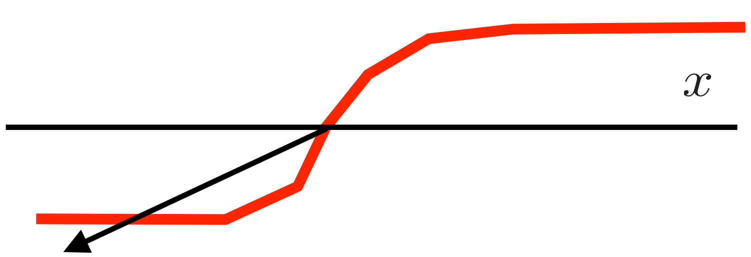
chiral mode!



$$\Delta E = k$$

Topology in electronic systems

Edge states Bulk-boundary correspondence



$$H_{\uparrow} = -i\sigma_1\partial_1 + \sigma_3 m(x) + \sigma_2 k$$
$$H_{\downarrow} = -i\sigma_1\partial_1 - \sigma_3 m(x) + \sigma_2 k$$

$\mathcal{T} - symmetry$

$$E = 0$$

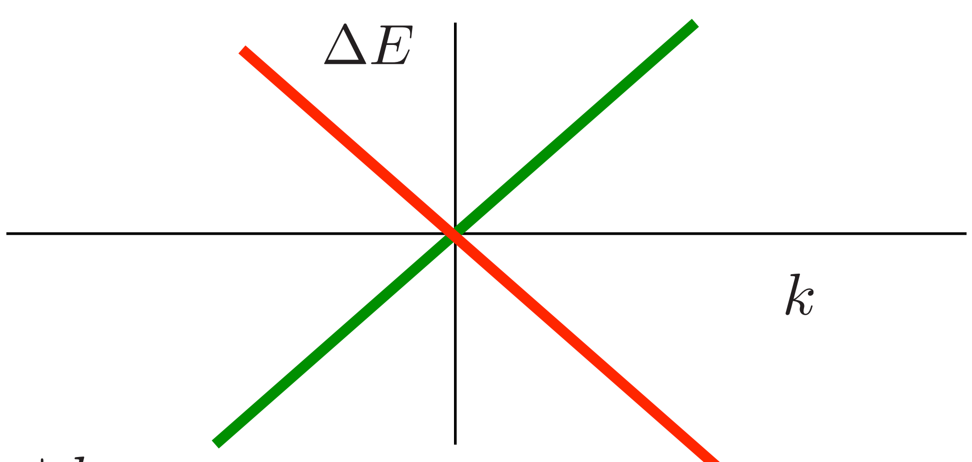
$$\psi = e^{iky} e^{-\int_x m(z)dz} a\,|s\rangle$$

$$E_{\boldsymbol{k}} = \pm\sqrt{\boldsymbol{k}^2 + m^2}$$

$$\psi_{\boldsymbol{k}}^{\alpha} = a_1^{\alpha}\,|1\rangle + a_{-1}^{\alpha}\,|-1\rangle$$

$$\sigma_r\,|s\rangle = s\,|s\rangle$$

helical mode!



$\Delta E = \pm k$

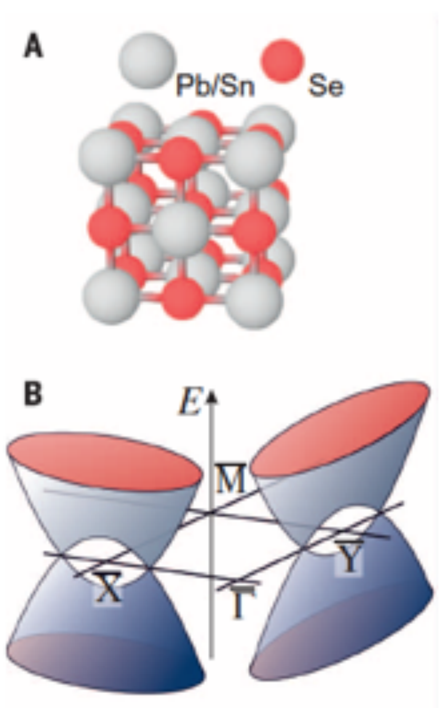
Topology in electronic systems

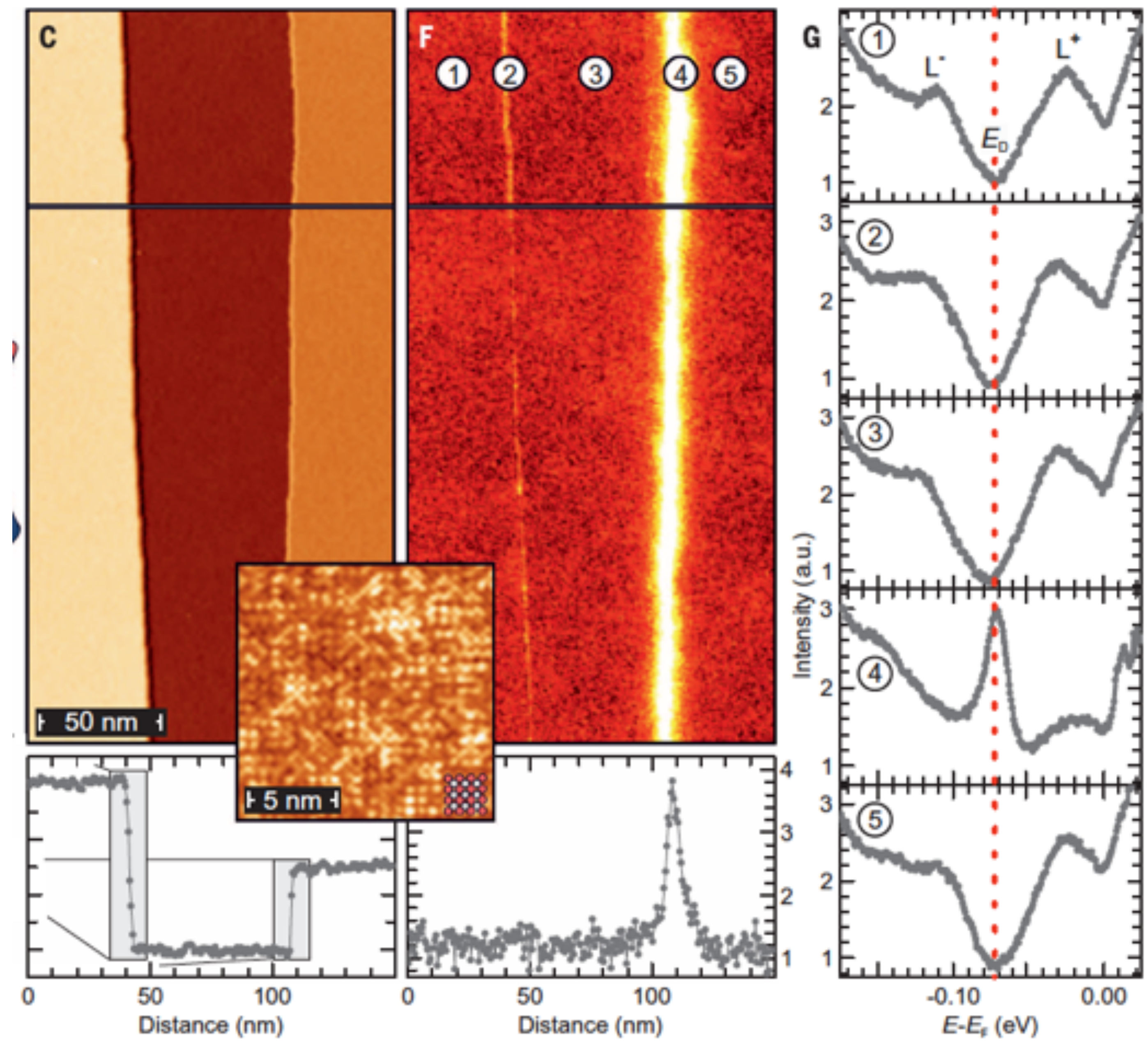
Edge states Bulk-boundary correspondence

TOPOLOGICAL MATTER

Robust spin-polarized midgap states at step edges of topological crystalline insulators

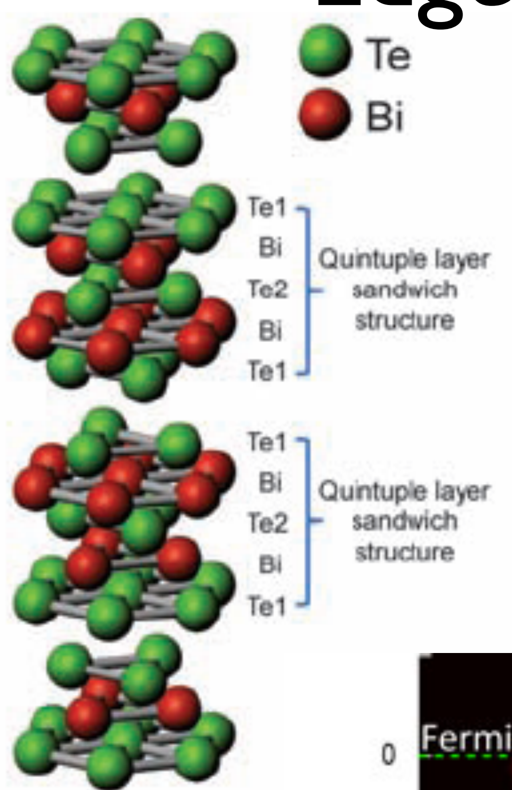
Paolo Sessi,[1*] Domenico Di Sante,[2] Andrzej Szczerbakow,[3] Florian Glott,[1]
Stefan Wilfert,[1] Henrik Schmidt,[1] Thomas Bathon,[1] Piotr Dziawa,[3] Martin Greiter,[2]
Titus Neupert,[4] Giorgio Sangiovanni,[2] Tomasz Story,[3]
Ronny Thomale,[2] Matthias Bode[1,5]



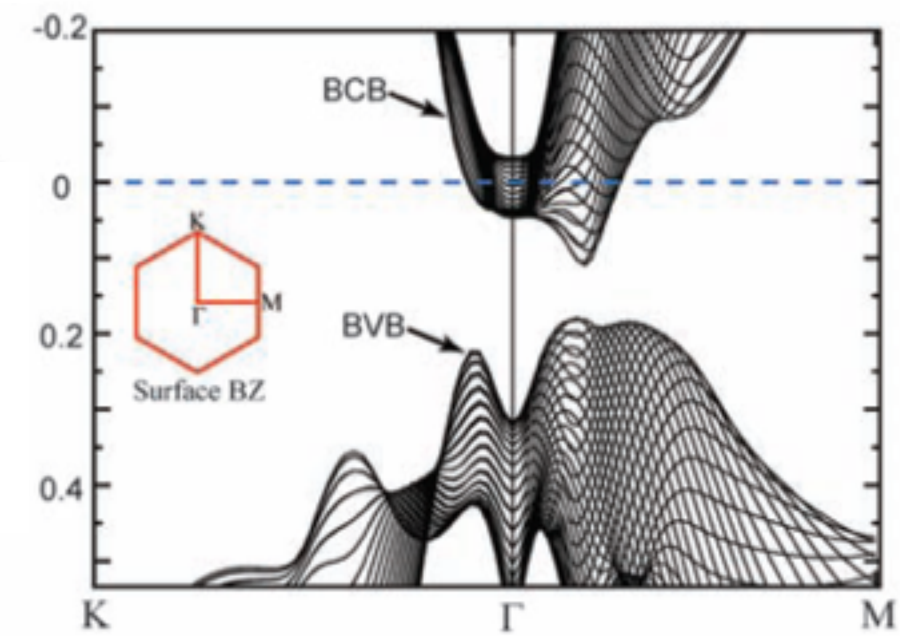


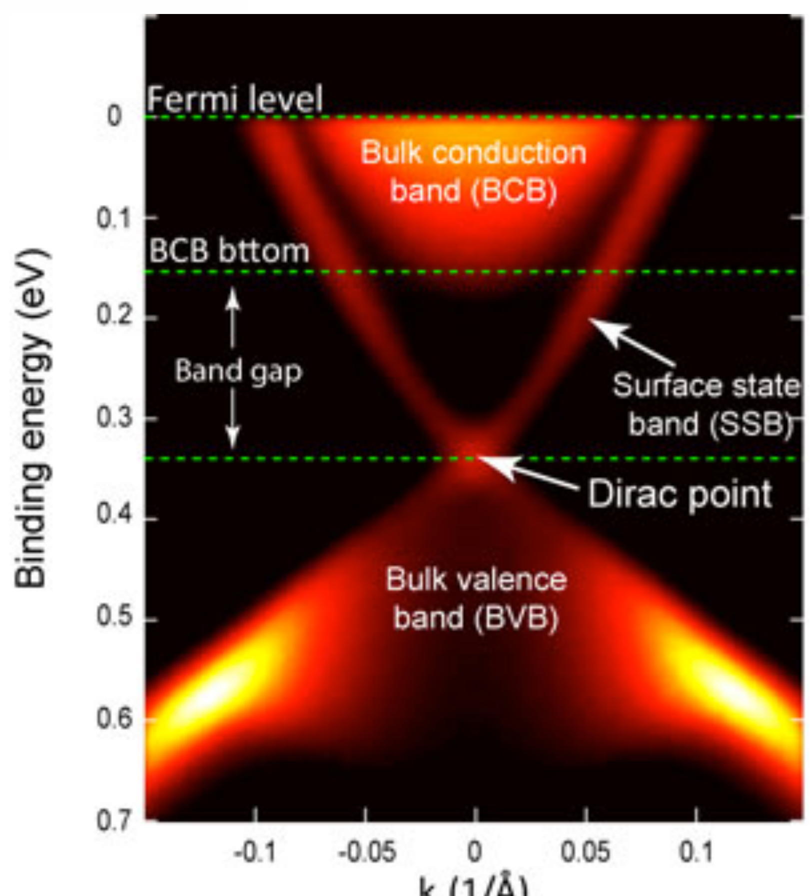
Topology in electronic systems

Edge states Bulk-boundary correspondence



$Bi_2Te_3 \quad Bi_2Se_3$

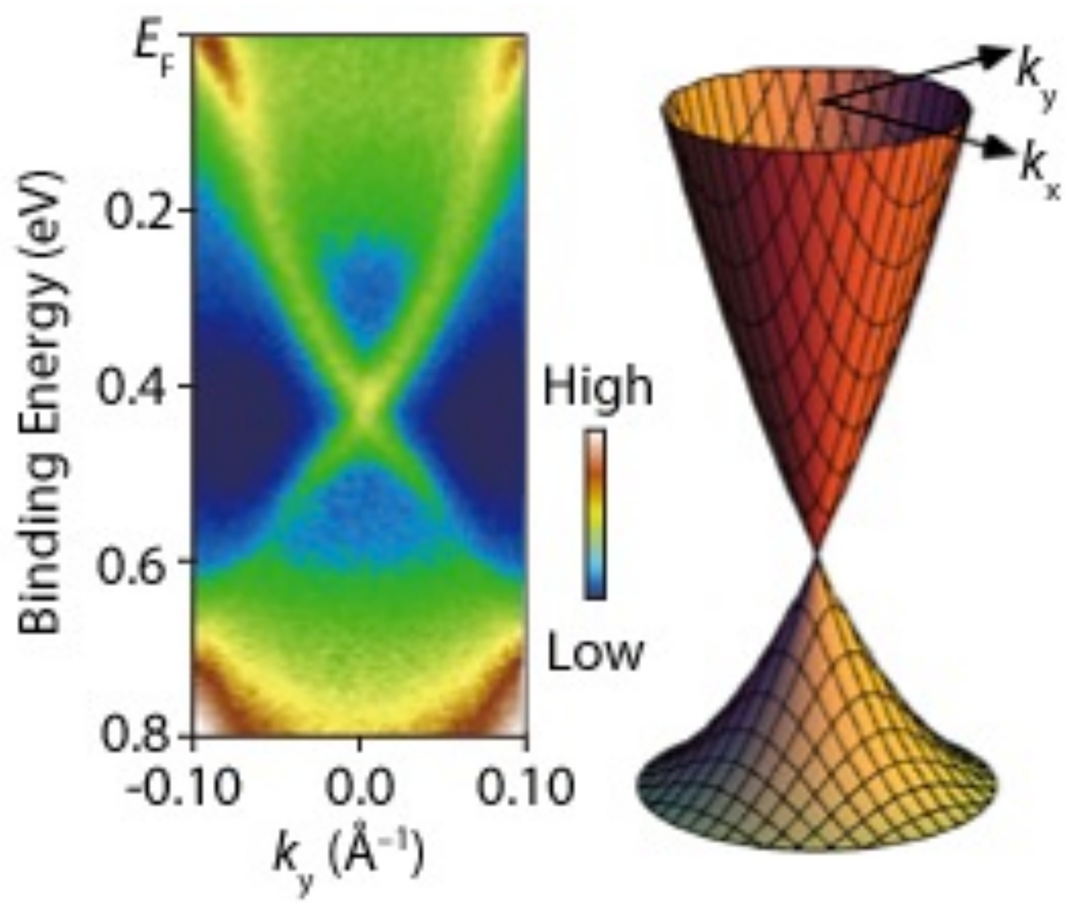




gapless surface state appears
Two dimensional analog
of the chiral/helical edge
states in 2D

Topology in electronic systems

Edge states Bulk-boundary correspondence



single specie massless Dirac fermion
(graphene: 4)

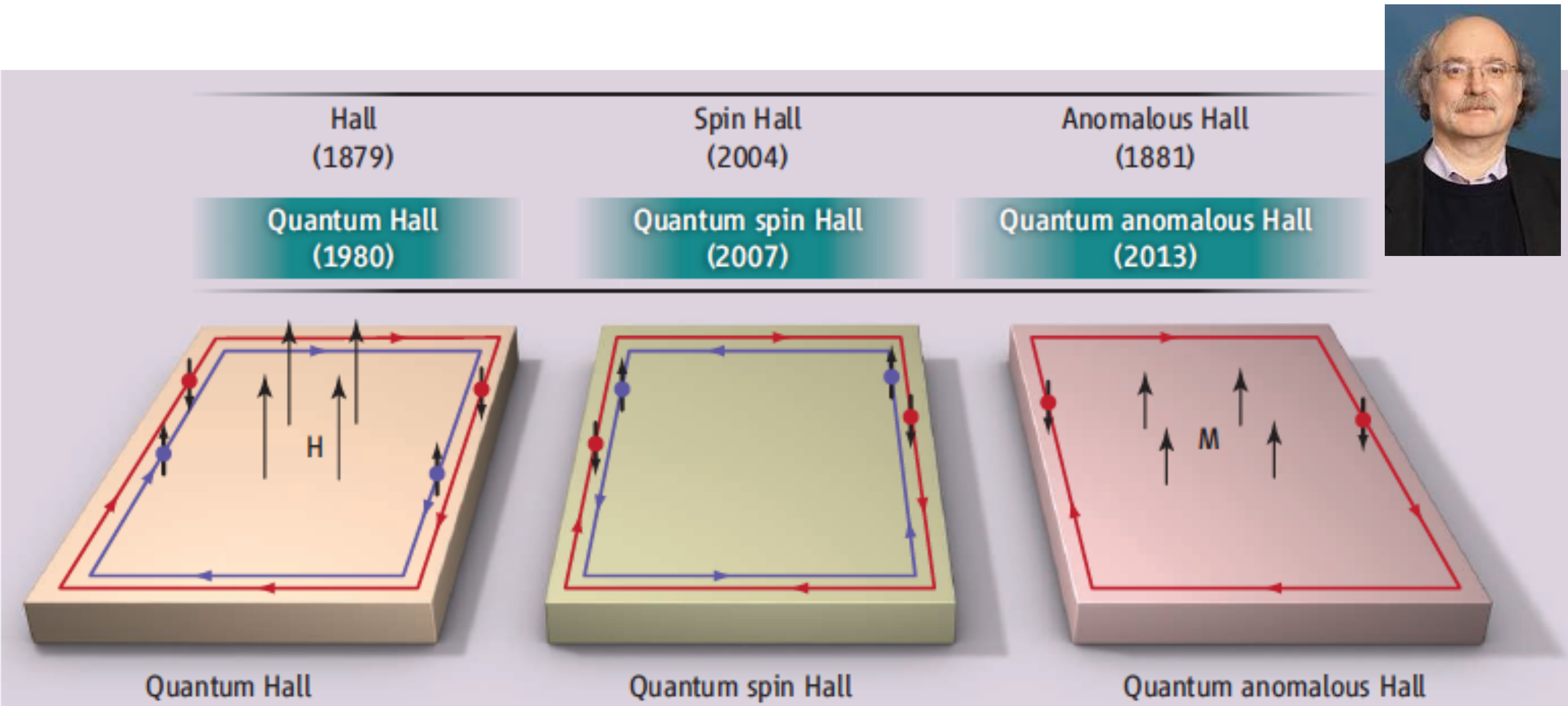
$$H_D = v_F \vec{n} \cdot \left(\vec{\sigma} \times \vec{\mathbf{k}} \right)$$

real spin

the direction of the spin is locked to the state's momentum

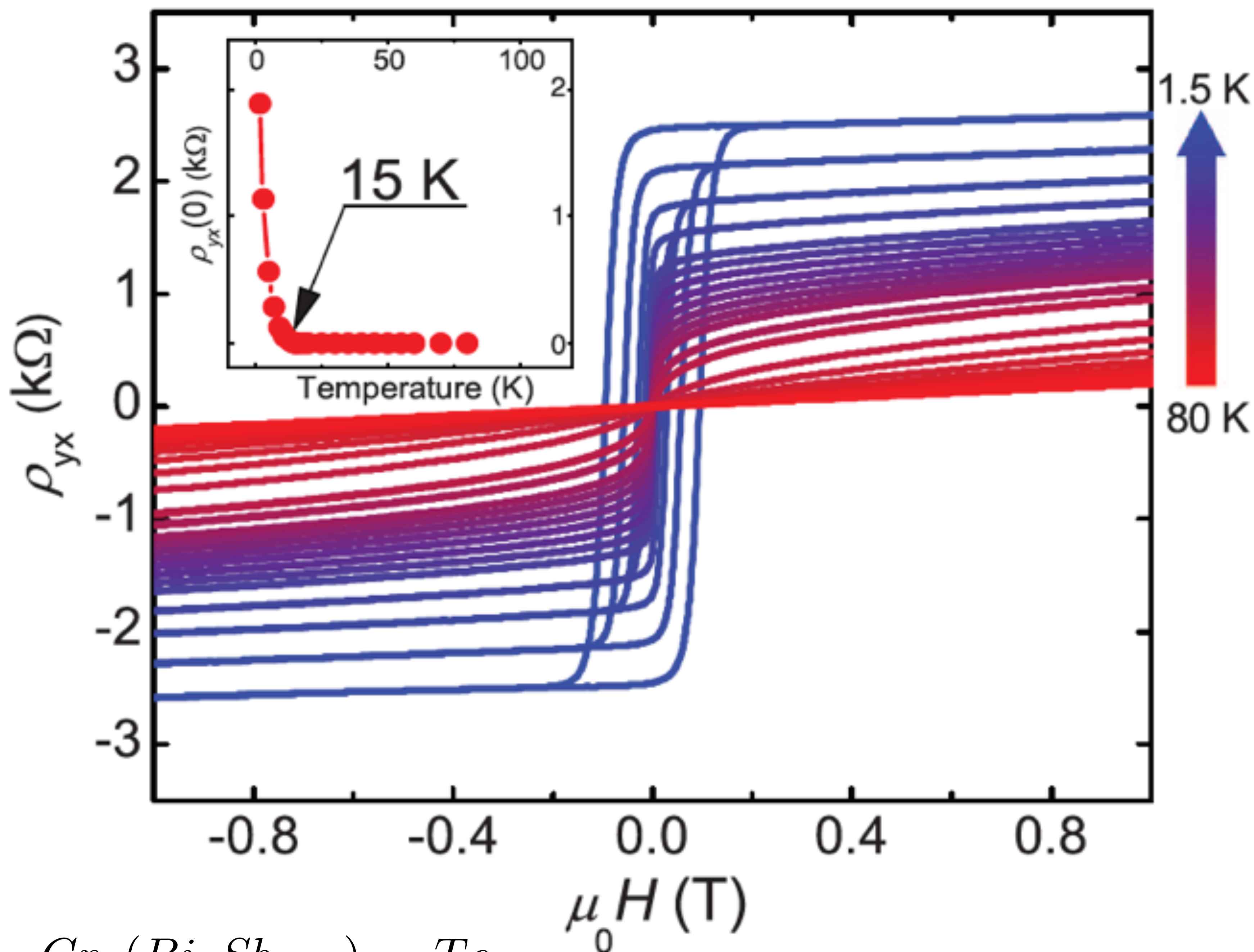
Where to look?

Quantum Anomalous Hall Effect (B=0) (2013!!)



Where to look?

Quantum Anomalous Hall Effect (B=0) (2013!!)



$Cr_x(Bi_ySb_{1-y})_{1-x}Te_3$

@B=0

$$\rho_{xy} \neq 0$$

magnetically doped
3DTI
with ordered
magnetization

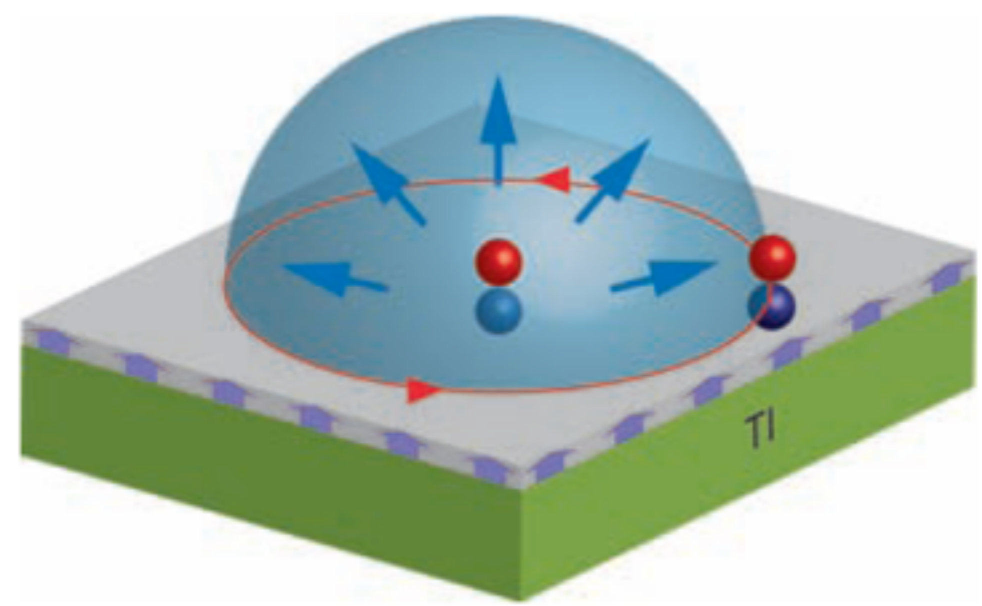
Where to look?

Three dimensional TI's!

REPORTS

Inducing a Magnetic Monopole with Topological Surface States

Xiao-Liang Qi,[1] Rundong Li,[1] Jiadong Zang,[2] Shou-Cheng Zhang[1]*



the system reacts to an external charge
as it had a monopole charge

Where to look?

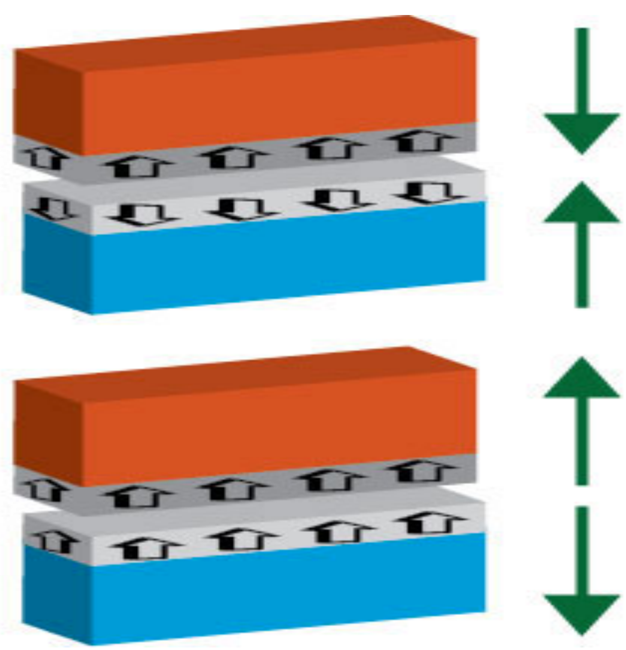
Three dimensional TI's!

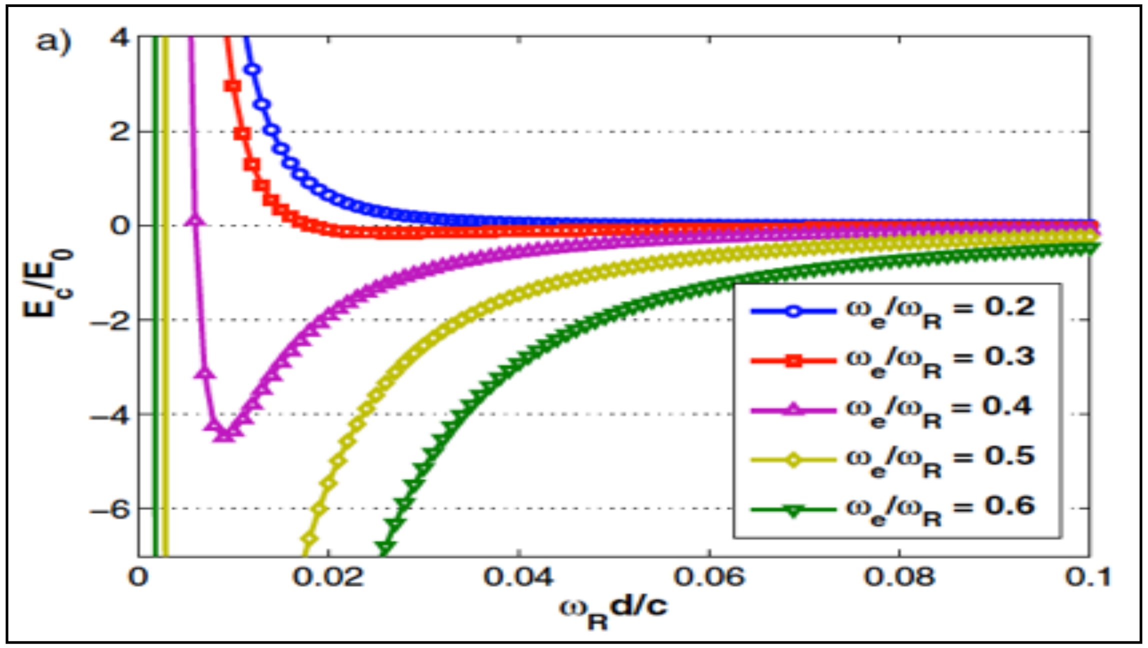
Possibility of repulsive Casimir effect!

PRL 106, 020403 (2011) PHYSICAL REVIEW LETTERS week ending 14 JANUARY 2011

Tunable Casimir Repulsion with Three-Dimensional Topological Insulators

Adolfo G. Grushin[1] and Alberto Cortijo[2]
[1]*Instituto de Ciencia de Materiales de Madrid (CSIC), Sor Juana Inés de la Cruz 3, Madrid 28049, Spain*
[2]*Department of Physics, Lancaster University, Lancaster, LA1 4YB, United Kingdom*
(Received 30 July 2010; revised manuscript received 13 December 2010; published 10 January 2011)





minimum=equilibrium=no force= levitation?

Where to look?

PRL **105,** 225901 (2010) PHYSICAL REVIEW LETTERS week ending 26 NOVEMBER 2010

Topological Nature of the Phonon Hall Effect

Lifa Zhang,[1] Jie Ren,[2,1] Jian-Sheng Wang,[1] and Baowen Li[2,1]
[1]*Department of Physics and Centre for Computational Science and Engineering, National University of Singapore, Singapore 117542, Republic of Singapore*
[2]*NUS Graduate School for Integrative Sciences and Engineering, Singapore 117456, Republic of Singapore*
(Received 2 August 2010; published 24 November 2010)

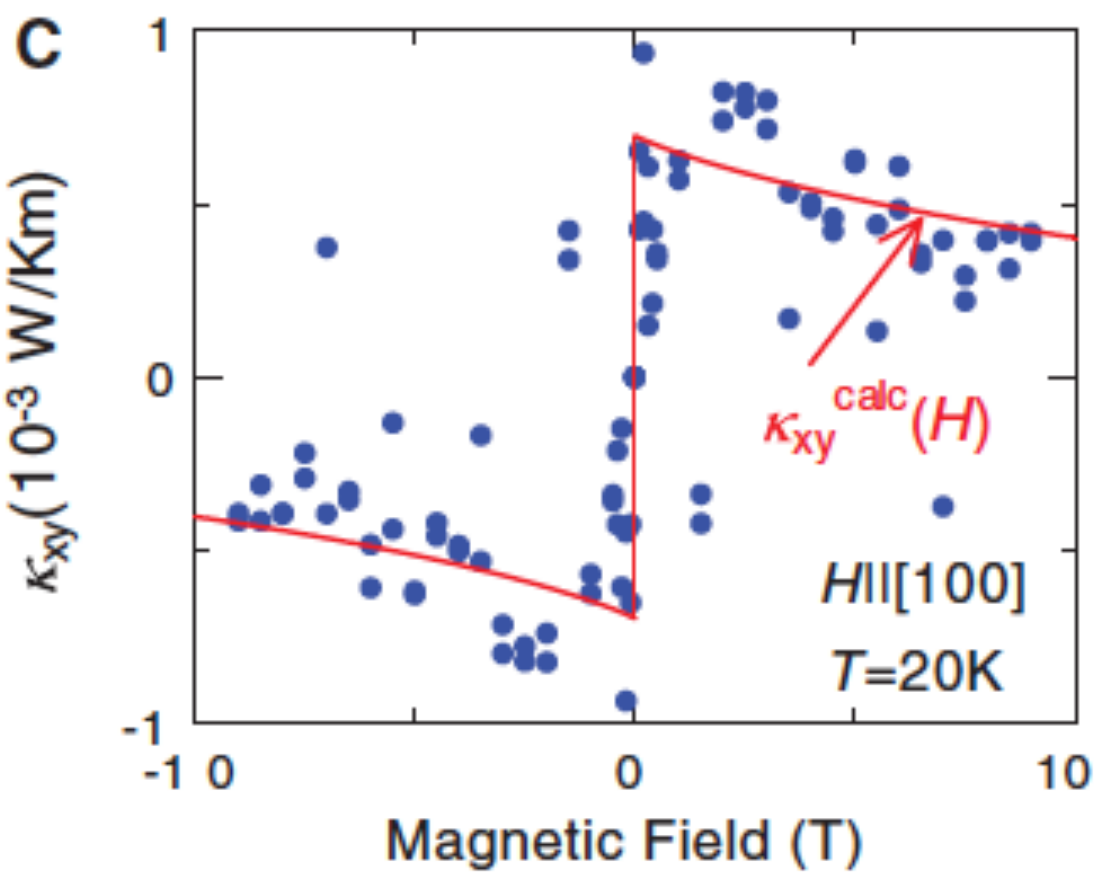
$$\kappa_{xy} = \frac{\hbar}{8VT} \sum_{\sigma \neq \sigma'} f(\omega_\sigma)(\omega_\sigma + \omega_{\sigma'})^2 \frac{i}{4\omega_\sigma \omega_{\sigma'}} \times \frac{\epsilon_\sigma^\dagger \frac{\partial D}{\partial k_x} \epsilon_{\sigma'} \epsilon_{\sigma'}^\dagger \frac{\partial D}{\partial k_y} \epsilon_\sigma - (k_x \leftrightarrow k_y)}{(\omega_\sigma - \omega_{\sigma'})^2},$$

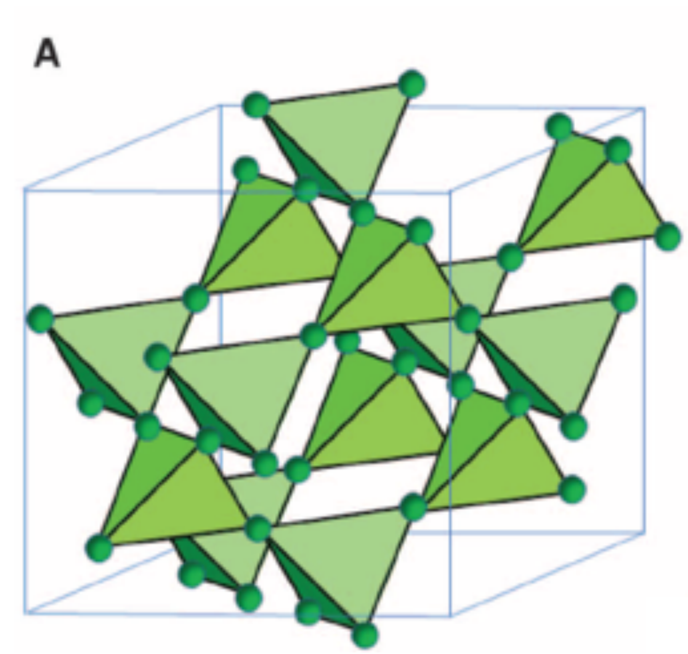
proportional to the Berry curvature of the phonon bands

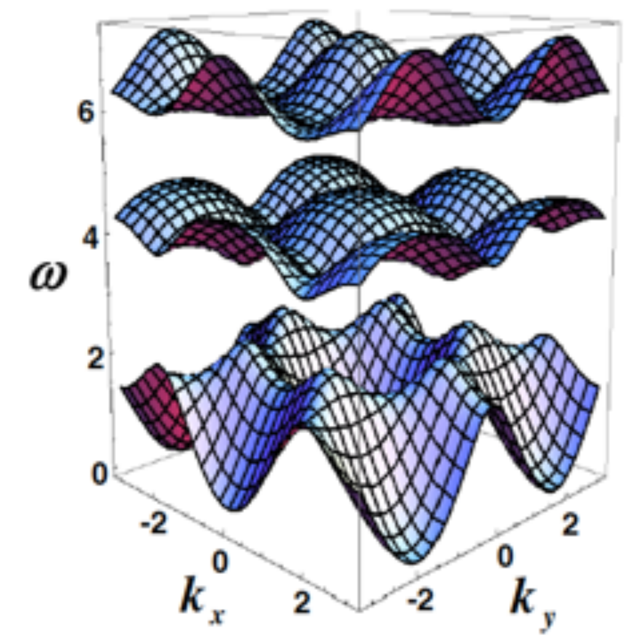
Where to look?

Observation of the Magnon Hall Effect

Y. Onose,[1,2*] T. Ideue,[1] H. Katsura,[3] Y. Shiomi,[1,4] N. Nagaosa,[1,4] Y. Tokura[1,2,4]







(b)

$$\kappa^{xy} \sim \frac{(6JS)^2}{2T} \int_0^\infty \frac{dk}{2\pi} \frac{k}{e^{\beta JSk^2} - 1} \left(\frac{\phi k^2}{27\sqrt{3}} \right) = \frac{\pi \phi}{36\sqrt{3}} T,$$

Where to look?

PRL **100,** 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA

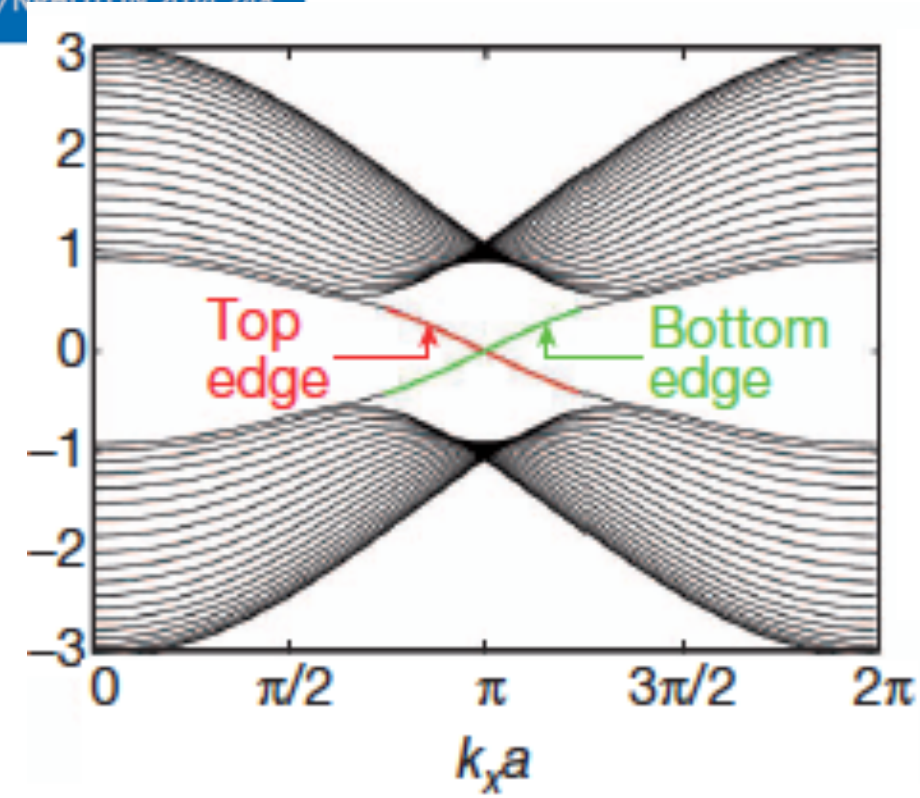
(Received 23 March 2005; revised manuscript received 30 May 2007; published 10 January 2008)

We show how, in principle, to construct analogs of quantum Hall edge states in "photonic crystals" made with nonreciprocal (Faraday-effect) media. These form "one-way waveguides" that allow electromagnetic energy to flow in one direction only.

nature photonics REVIEW ARTICLE
PUBLISHED ONLINE: 26 OCTOBER 2014 | DOI: 10.1038/N

Topological photonics

Ling Lu*, John D. Joannopoulos and Marin Soljačić



Where to look?

LETTER

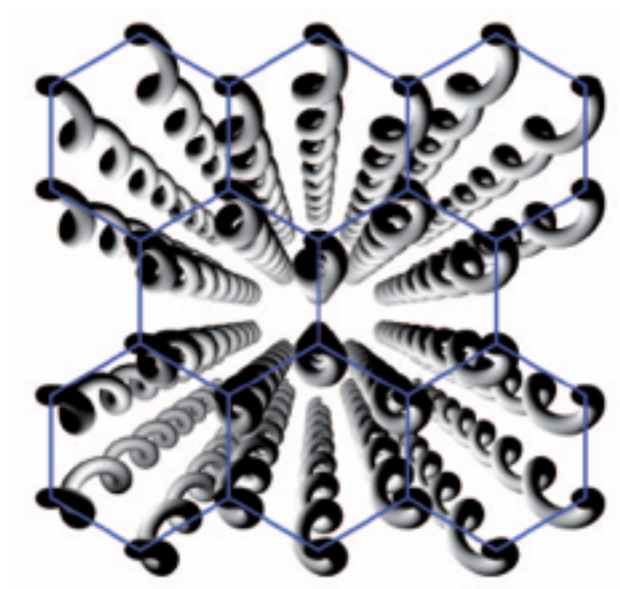
doi:10.1038/nature12066

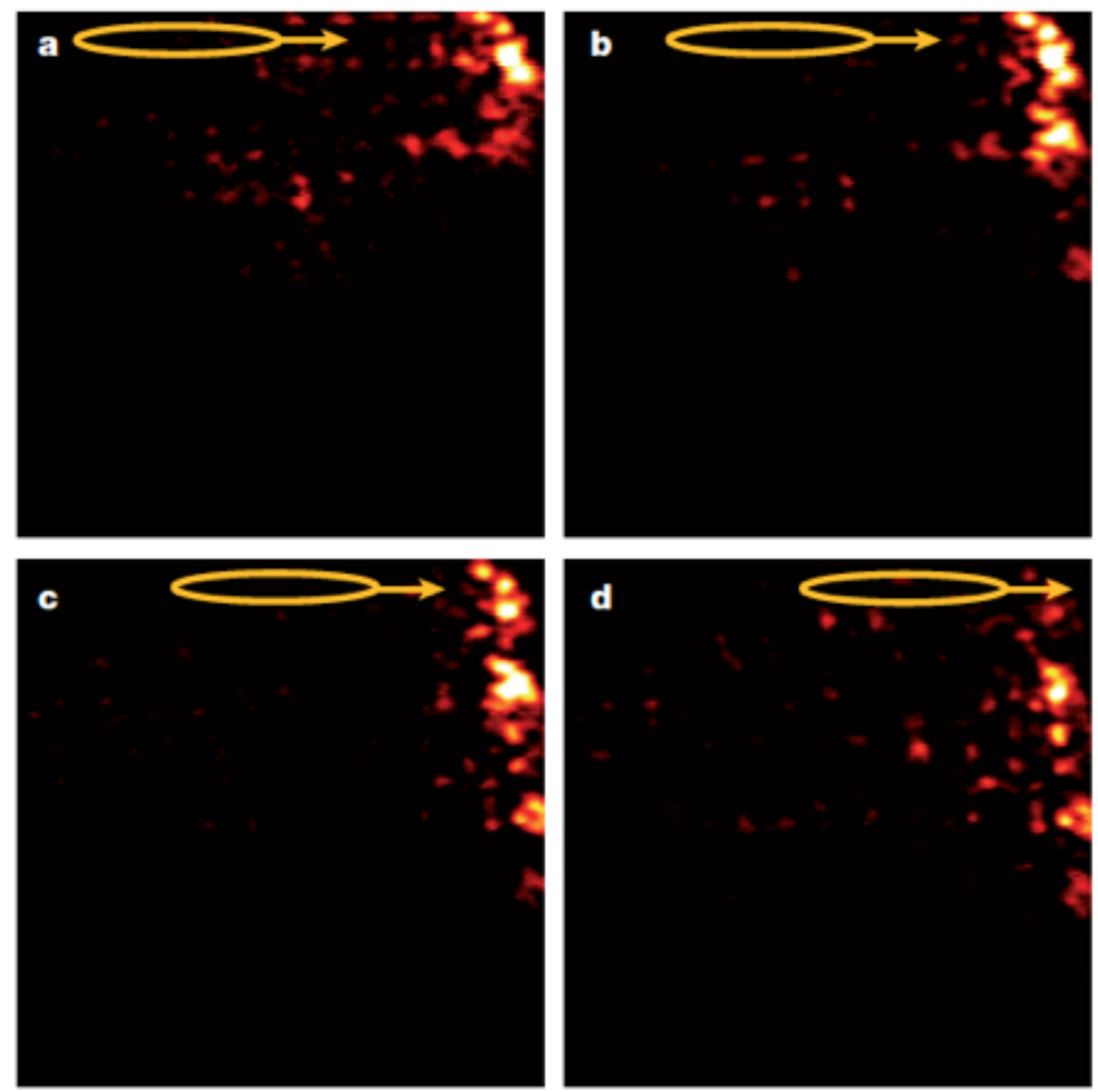
Photonic Floquet topological insulators

Mikael C. Rechtsman[1]*, Julia M. Zeuner[2]*, Yonatan Plotnik[1]*, Yaakov Lumer[1], Daniel Podolsky[1], Felix Dreisow[2], Stefan Nolte[2], Mordechai Segev[1] & Alexander Szameit[2]

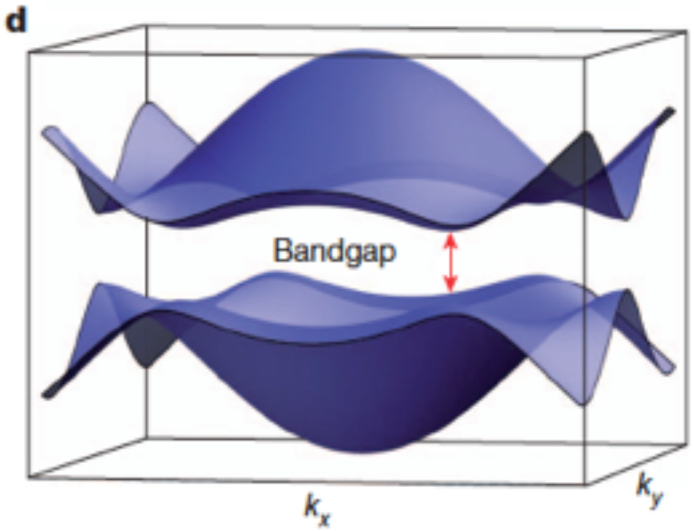
edge states made of light!

b





d



Where to look?

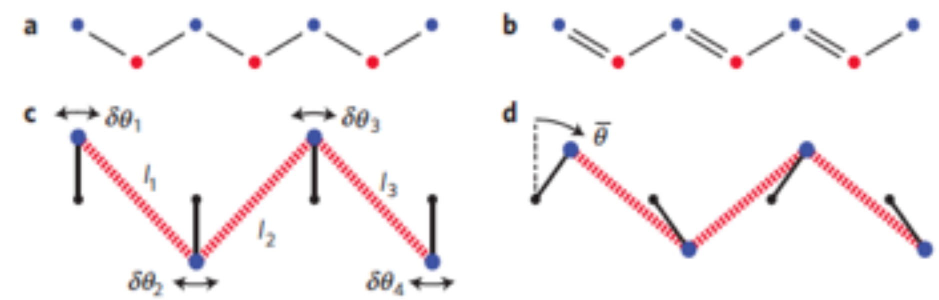
nature physics

ARTICLES

PUBLISHED ONLINE: 8 DECEMBER 2013 | DOI: 10.1038/NPHYS2835

Topological boundary modes in isostatic lattices

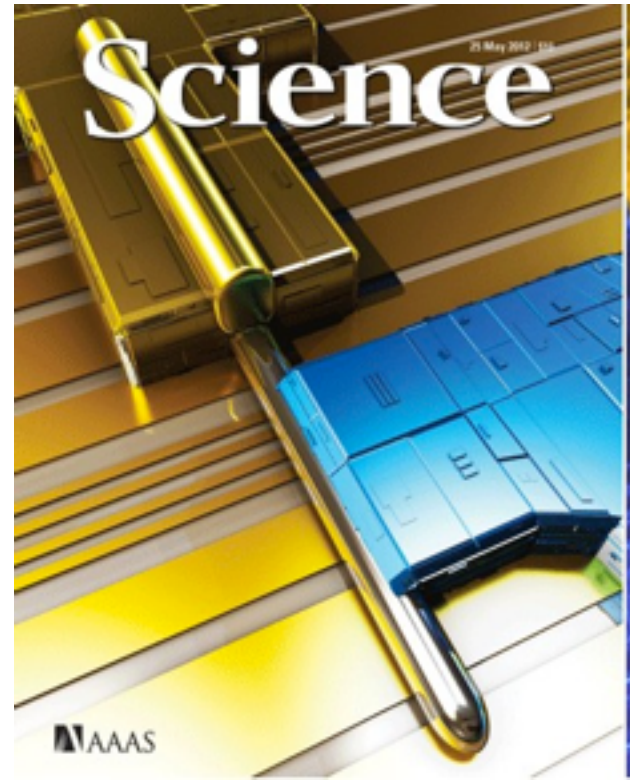
C. L. Kane and T. C. Lubensky*

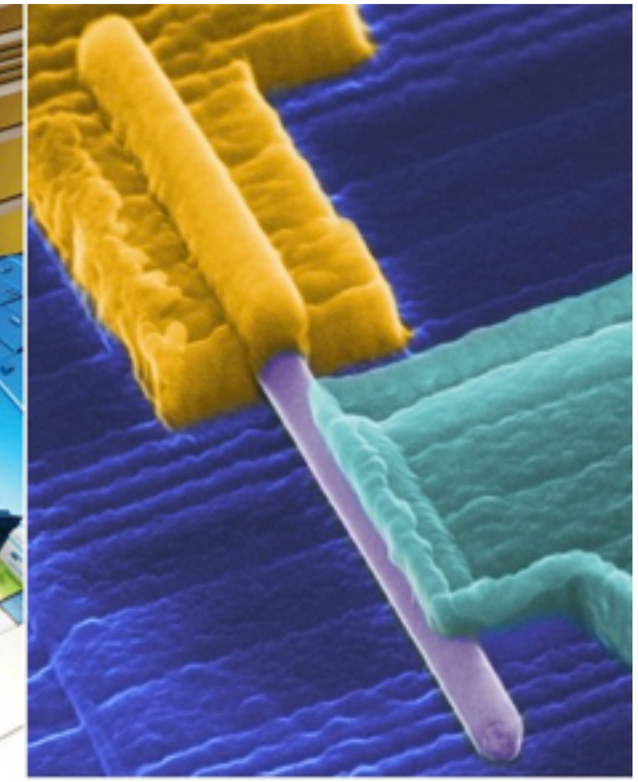


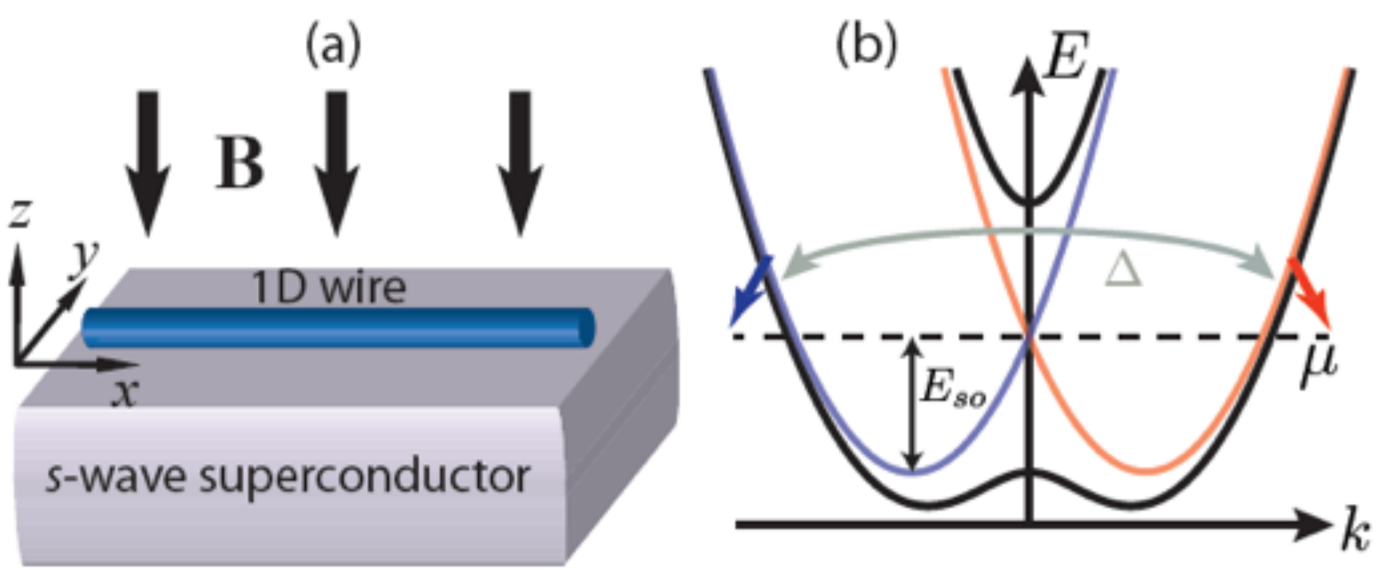
Where to look?

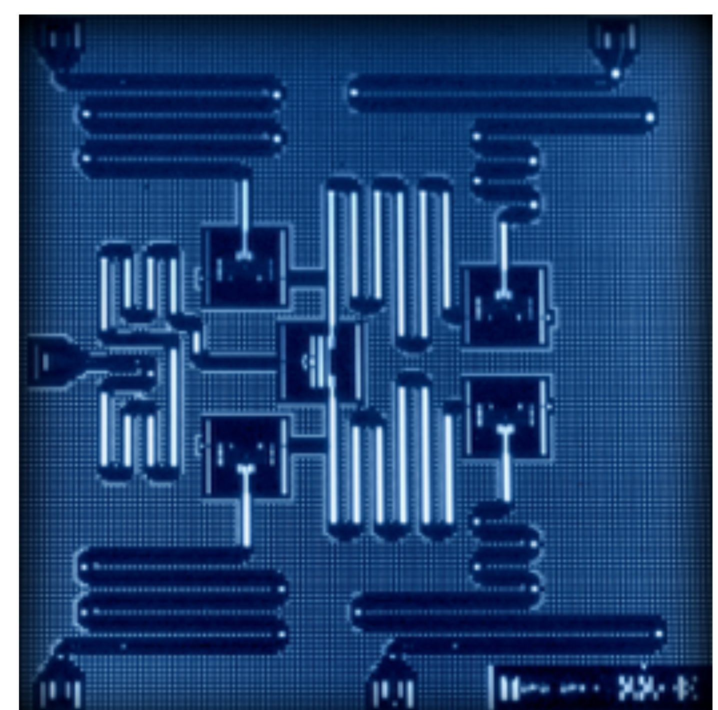
Majorana zero modes:

$$\gamma_0 = \gamma_0^+$$





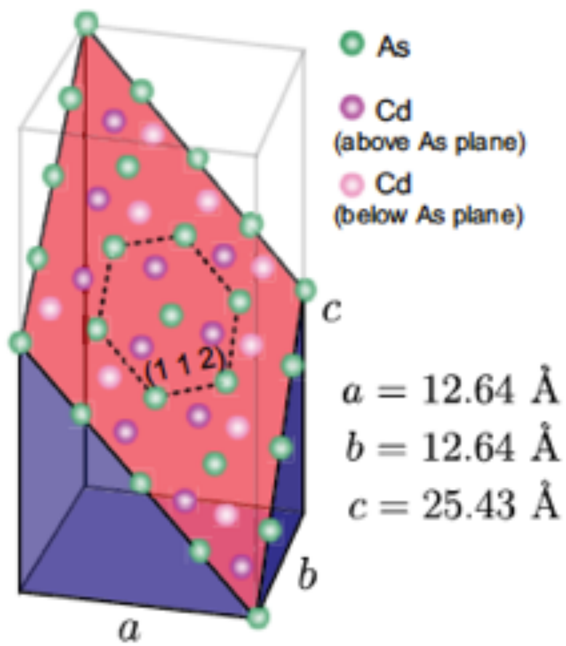


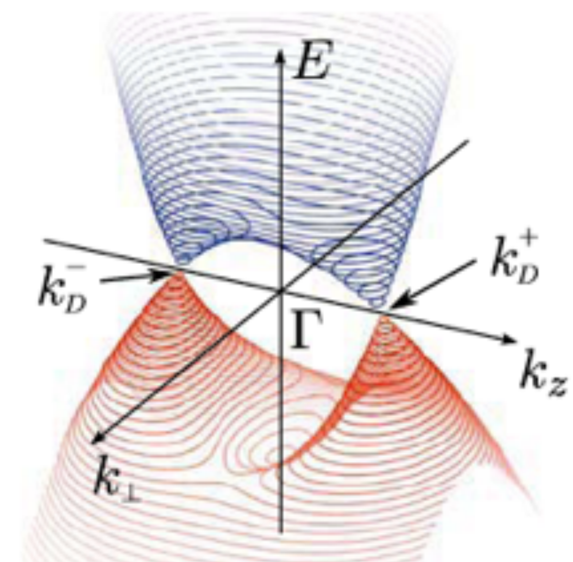


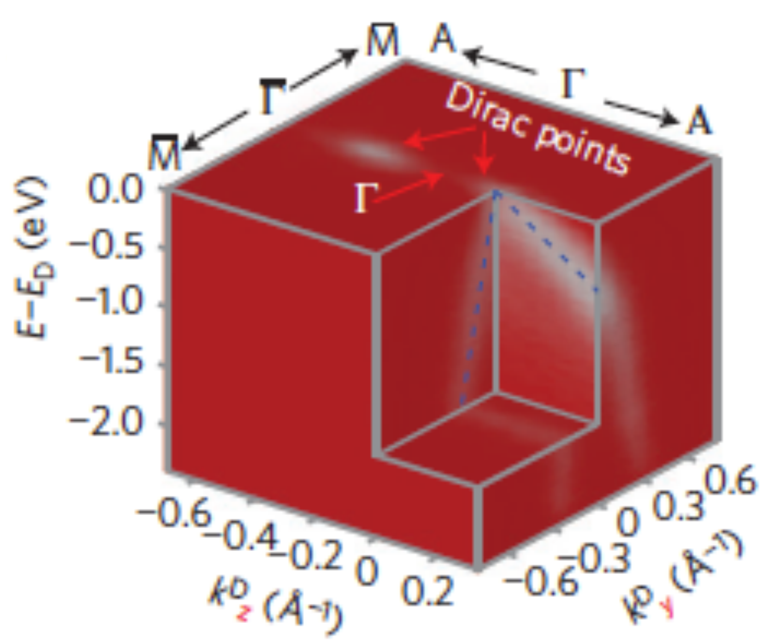
?

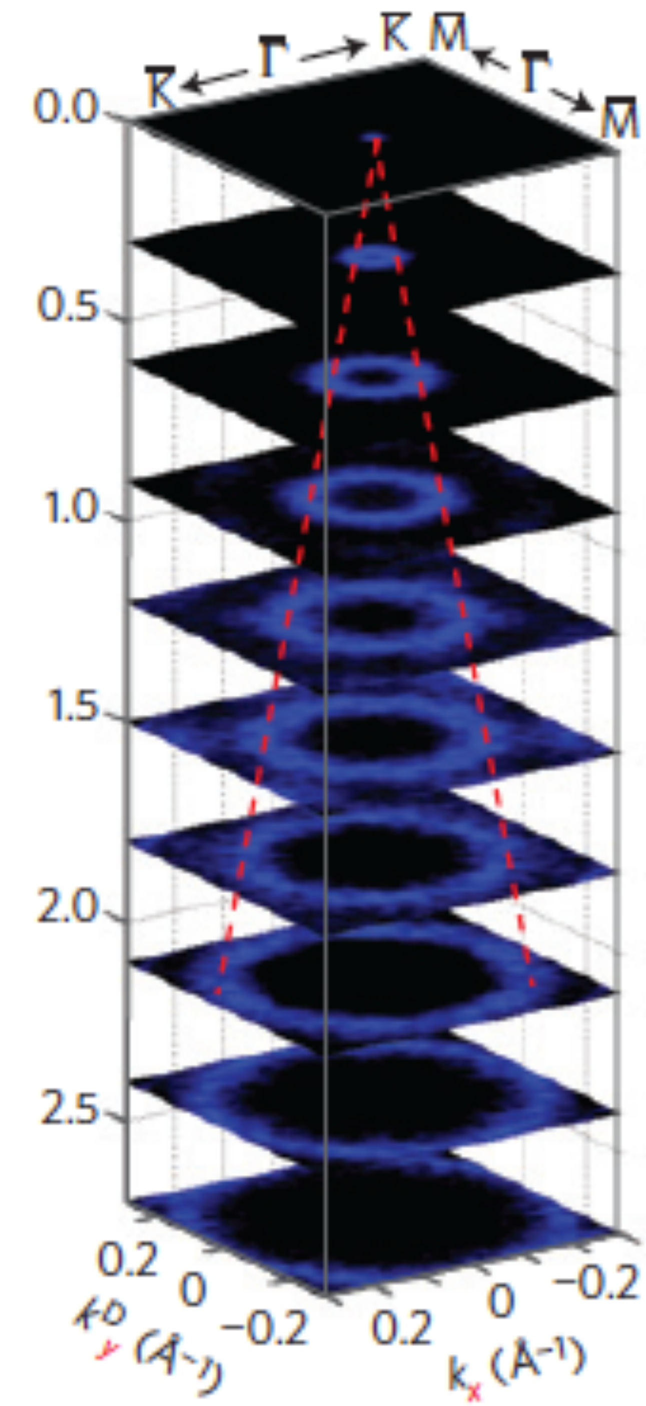
Where to look?

Weyl semimetals







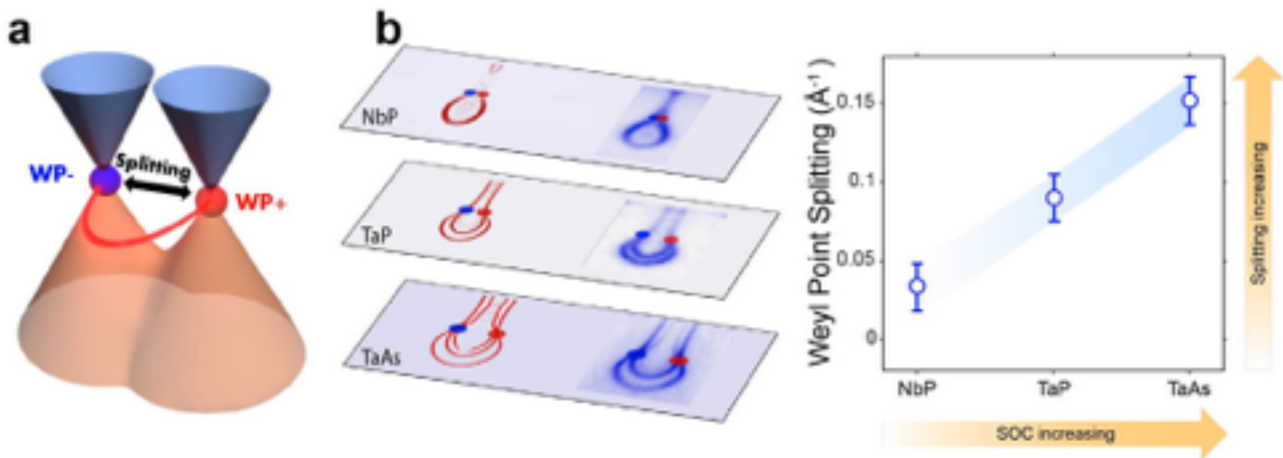


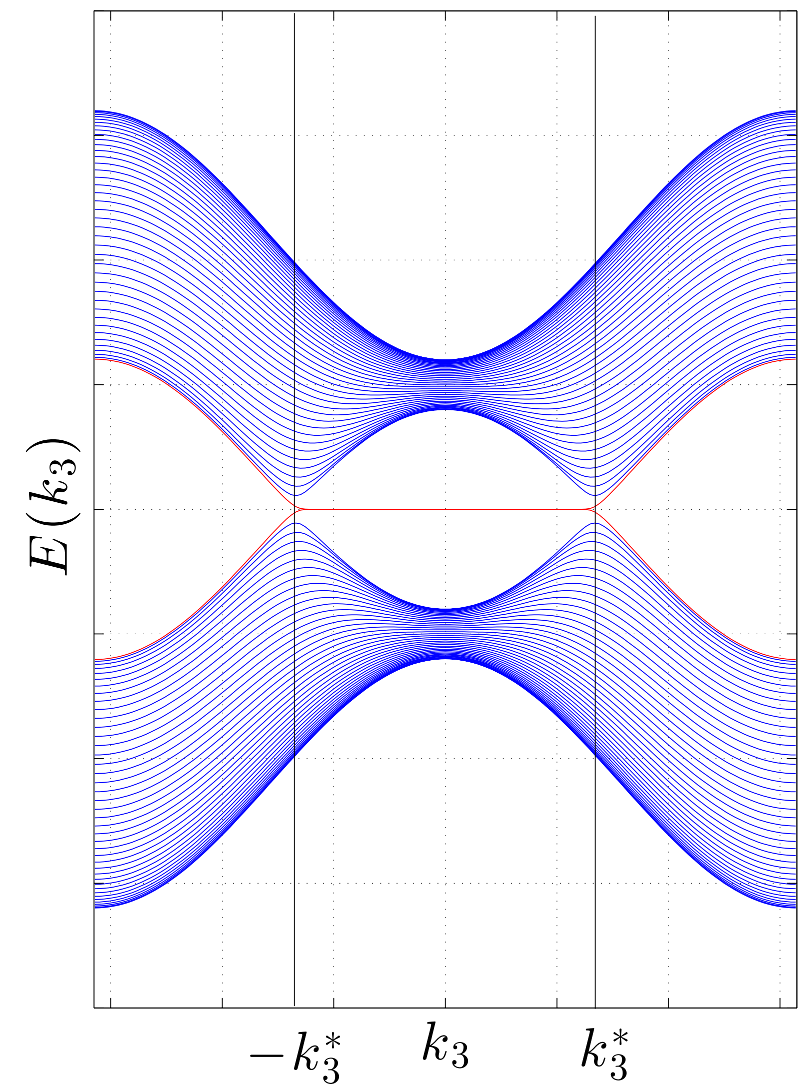
Dirac points away from high symmetry points

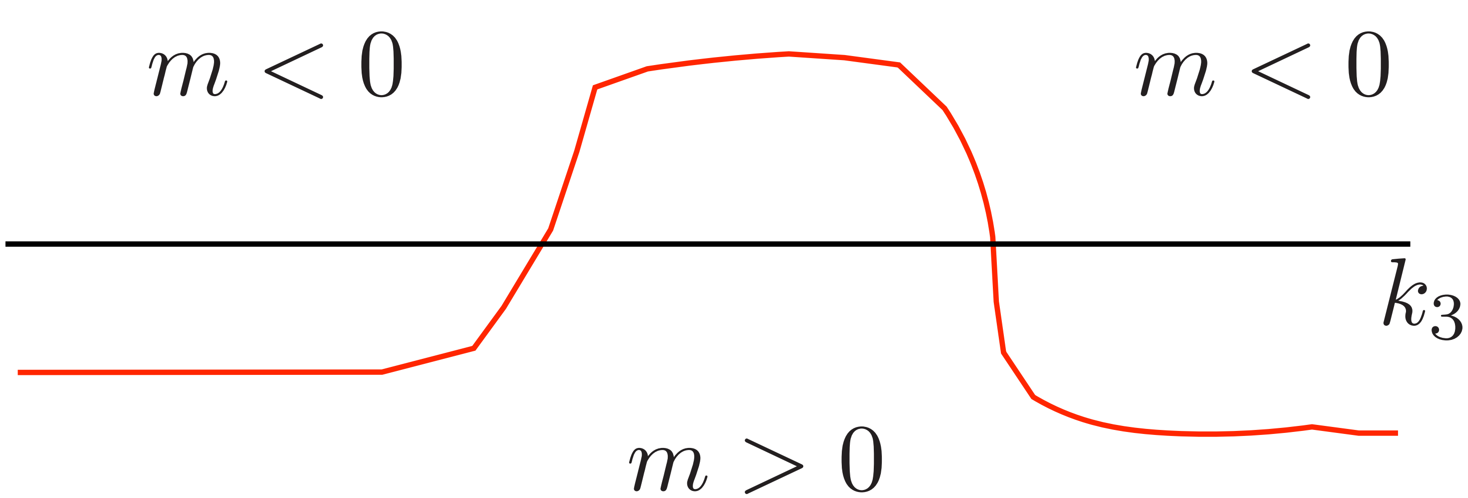
Where to look?

Weyl semimetals

Fermi arcs







$$m(x) \to m(x, k_3)$$

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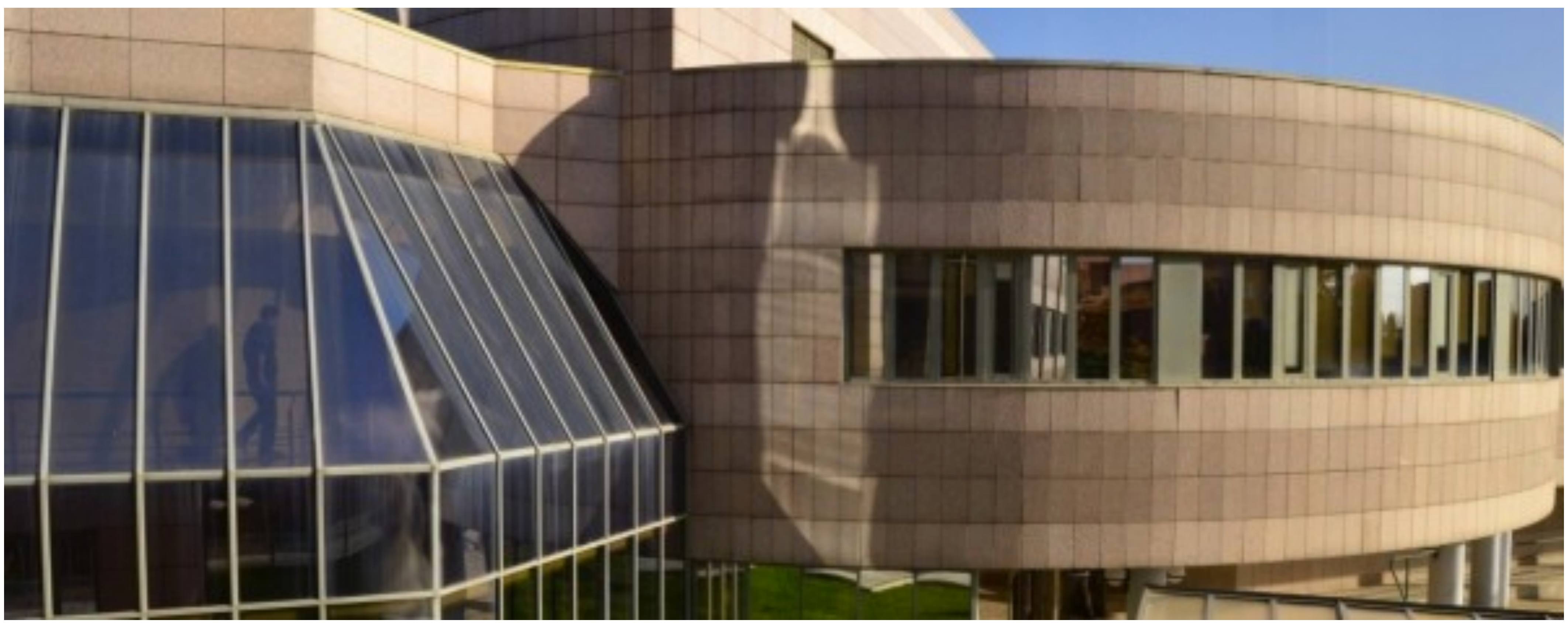

Thank you for your attention!!